MOCK CET – 3
PHYSICS, CHEMISTRY & MATHEMATICS

ANSWER KEYS

ANSWERS												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
4	1	3	4	1	1	2	3	4	1	2	2	1
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
2	3	3	3	2	2	1	3	2	2	4	1	3
27	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
4	1	3	1	1	2	1	2	1	4	2	1	1
40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**
4	4	3	2	4	2	4	2	4	1	1	3	4
53	**54**	**55**	**56**	**57**	**58**	**59**	**60**					
4	2	1	1	2	3	1	1					

HINTS & SOLUTIONS

1. Here, $v = \frac{ds}{dt} = 3t^2 - 12t + 3$;
   $\Rightarrow a = \frac{dv}{dt} = 6t - 12$
   For $a = 0, 6t - 12 = 0$
   $\Rightarrow t = 2$
   At $t = 2s, v = 12 - 24 + 3 = -9ms^{-1}$
2. Form the equation of motion, $v^2 = u^2 - 2as$;
   When $v = 0$, then the equation becomes
   $u^2 = 2as$
   $\Rightarrow s \propto u^2$
   $\Rightarrow \frac{S_1}{S_2} = \frac{u_1^2}{u_2^2}$
   $\Rightarrow \frac{2}{S_2} = \frac{(40 \times 5/18)^2}{(80 \times 5/18)^2}$
   $S_2 = 8$ meters
3. Here, $x = at^2$ *and* $v_x = \frac{dx}{dt} = 2at$;
   $v_y = \frac{dy}{dt} = \frac{d}{dt}(bt^2) = 2bt$
   $\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{4a^2t^2 + 4b^2t^2} = 2t\sqrt{a^2 + b^2}$
4. Kinetic energy acquired by particle $= \frac{1}{2}mv^2$
   Since particle travels distance d from rest then applying $v^2 = u^2 - 2as$
   $\Rightarrow v^2 = 2ad\ (since\ u = 0)$
   $\Rightarrow v^2 = 2\frac{F}{m}d\ (since\ F\ is\ constant)$
   Hence K.E $= \frac{1}{2}m.\frac{2F}{m}d = Fd$
5. According to Newton's first law of motion, the ball will revolve around the earth as no external force is applied.
6. According to conservation towards right, then
   $(m_1 + m_2)v = m_1u_1 + m_2u_2$
   $\Rightarrow (m + 2m)v = m \times 3$
   $\Rightarrow v = 1km/hour$

7. Kinetic energy $= \frac{1}{2}mv^2$

   Again, $v^2 = u^2 + 2as = 0 + 2ad$

   Or $v^2 = 2\left(\frac{F}{m}\right)d$

   $\therefore KE = \frac{1}{2}m \times 2\frac{F}{m}d = Fd$

   $\therefore KE \propto d$

8. Given : $\vec{F} = (-2\hat{\imath} + 15\hat{\jmath} + 6\hat{k})N$,

   $\vec{d} = 10\hat{\jmath}\,m$

   Work done, $W = \vec{F}.\vec{d}$

   $= (-2\hat{\imath} + 15\hat{\jmath} + 6\hat{k}).(10\hat{\jmath})$

   =150J

9. Centripetal force, $F = mv_{max}^2$

   $25 = \frac{0.25 \times v_{max}^2}{1.96}$

   $\Rightarrow v_{max}^2 = \frac{25\times1.96}{0.25} = 196$

   $\Rightarrow v_{max} = 14m/s$

10. Given : $\vec{F} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$

    $\vec{r} = (7\hat{\imath} + 3\hat{\jmath} + \hat{k})$

    Now, $\vec{\tau} = \vec{r} \times \vec{F}$

    $\therefore \vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$

    $= \hat{\imath}(15-1) - \hat{\jmath}(35+3) + \hat{k}(7+9)$

    $= 14\hat{\imath} - 38\hat{\jmath} + 16\hat{k}$

11. Escape velocity, $v_e = \sqrt{\frac{2GM}{R}} = 11.2km/sec$

    And $v'_e = \sqrt{\frac{2GM}{R/4}} = 2\sqrt{\frac{2GM}{R}}$

    $\Rightarrow v'_e = 2v_e$

    $= 22.4km/sec$

12. Self explanatory

13. $T \propto \frac{1}{\sqrt{g}}$

    Inside a satellite, $g = 0$

    Hence period will be infinite

14. Self explanatory

15. Here, $v^2 = \omega^2(r^2 - y^2)$

    $\therefore 9 = \omega^2(r^2 - 16)$,

    And $16 = \omega^2(r^2 - 9)$

    Dividing, we get $\frac{9}{16} = \frac{r^2-16}{r^2-9}$

    Or $7r^2 = 175$

    Or $r = 5m$

16. Given: $l_1 = 0.5m, l_2 = 2.0m$

    Time period of simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

    For $l_1 = 0.5m, T_1 = 2\pi\sqrt{\frac{0.5}{g}}$

    And for $l_2 = 2.0m\ T_2 = 2\pi\sqrt{\frac{20}{g}}$

    $\Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{20}{0.5}} = 2$

    $\Rightarrow T_2 = 2\,T_1$

Hence both the pendulums will be in same phase when the shorter length pendulum completes 2 oscillations.

17. Phase difference $= \frac{2\pi}{\lambda} \times path\ difference$

$\therefore \frac{\pi}{3} = \frac{2\pi}{\lambda} \times path\ difference$

$\Rightarrow$ Path difference $= \frac{\lambda}{6}$

18. Doppler effect is not observed when source and observer are moving in perpendicular direction.
19. Self explanatory
20. Efficiency of carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

In case I $0.4 = 1 - \frac{T_2}{500}$

$\Rightarrow \frac{T_2}{500} = 1 - 0.4$

$\Rightarrow T_2 = 500 \times 0.6 = 300^0K$

In case II $0.5 = 1 - \frac{300}{T'_1}$

$\Rightarrow \frac{300}{T'_1} = 1 - 0.5$

$\Rightarrow T'_1 = \frac{300}{0.5} = 600^0K$

21. Mass and volume of the gas will remain same, so density will also remain same.
22. According to Newton's law of cooling if two liquids are cooled under identical conditions, then their rates of cooling will be same.
23. According to Wien's displacement law

$\lambda_m = constant$

$\Rightarrow \lambda_m \propto \frac{1}{T}$

$\therefore \frac{(\lambda_m)'}{\lambda_m} = \frac{T_1}{T_2} = \frac{2000}{3000}$

Or $(\lambda_m) = \frac{2}{3}\lambda_m$

24. Resultant force $= \sqrt{F^2 + F^2 + 2F^2 cos60^0}$

$= \sqrt{3F^2} = \sqrt{3}F$

25. $V = 5x^2 + 10x - 9$

Differentiating, we get $\frac{dV}{dx} = 10x + 10$

At point $x = 1; \frac{dV}{dx} = 20$

But $\frac{dV}{dx} = E, therefore\ E = 20Vm^{-1}$

26. There will be two dipoles inclined to each other at an angle of 60^0. The dipole moment of each dipole will be (ql)

$\therefore$ Resultant dipole moment

$= \sqrt{(ql)^2 + (ql)^2 + 2(\text{ql})(\text{ql})\cos 60^0} = \sqrt{3}(ql)$

27. $R = \rho\frac{l}{\pi r^2}\ and\ m = \pi r^2 ld$

$\therefore R \propto \frac{l^2}{m}$

$\therefore R_1 : R_2 : R_3 = \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$

28. $V_d = \frac{I}{neA}$

$= \frac{1}{8 \times 10^{22} \times 1.6 \times 10^{-19} \times 7.81 \times 10^6}$

$= 10m/s$

$\therefore \frac{l}{V_d} = \frac{10 \times 10^{-1}}{10} = 10^{-2}sec$

29. Self explanatory
30. Drift velocity, $v_d = \frac{I}{neA} = \frac{V}{neAR}$

Or $v_d \propto V$

So, when the potential difference is doubled, the drift velocity will be doubled.

31. $F = Bil\sin\theta$
$= 2 \times 0.5 \times 5 \times 10^{-4} \times \sin 60^0$
$= 4.33 \times 10^{-4}$N

32. Here $\frac{mv^2}{r} = Bqv$
Or $v = \frac{Bqr}{m}$
Or $r\omega = \frac{Bqr}{m}$
Or $\omega = \frac{Bq}{m}$
Or $\frac{2\pi}{T} = \frac{Bq}{m}$
Or $T = \frac{2\pi m}{Bq}$
But $T \propto \frac{m}{q}$
$\therefore \frac{T_\alpha}{T_p} = \frac{m_\alpha}{M_p} \times \frac{q_p}{q_\alpha} = 4\frac{m_p}{m_p} \times \frac{q_p}{2q_p} = 2$

33. Self explanatory

34. Charge in capacitor, $q = CV$
$= 10^{-6} \times 200\sqrt{2}\sin 100t$
$or\ i = \frac{dq}{dt} = 10^{-6} \times 200\sqrt{2} \times 100\cos 100$
$t = 20\sqrt{2} \times 10^{-3}\cos 100\,t$
∴ Ammeter reading in rms value
$= \frac{I_m}{\sqrt{2}} = 20\sqrt{2} \times \frac{10^{-3}}{\sqrt{2}} = 20mA$

35. $X_c = \frac{1}{\omega C}$
$or\ f = \frac{1}{2\pi C X_c}$
$= 1 \times \frac{1000}{6.28 \times 5 \times 10^{-6}}$
$= \frac{1000}{\pi} MHz$

36. Charge, $q = \omega CV = \omega CV_0\ sin\omega t$
$\therefore i = \frac{dq}{dt} = \omega CV_0 \cos\omega t = \omega CV_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

37. Phase difference, $\emptyset = tan^{-1}\frac{\omega L}{R}$
$= tan^{-1}\frac{2\pi \times 50 \times 1.4}{440} = tan^{-1}1 = \frac{\pi}{4}$
$\therefore \omega t = \frac{\pi}{4}$
$or\ t = \frac{\pi}{4} \times \frac{1}{\omega} = \frac{\pi}{4} \times \frac{1}{2\pi \times 50} = 2.5 \times 10^{-3} sec$

38. Force due to current carrying wire
$F = i(dl \times B) = ilB\sin\theta$
Or $\sin\theta = \frac{F}{ilB} = \frac{15}{1.5 \times 10 \times 2} = \frac{1}{2}$
Or $\theta = sin^{-1}\left(\frac{1}{2}\right) = 30^0$

39. Distance between two consecutive dark fringes
$= \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 1}{0.6 \times 10^{-3}}$
$= 1 \times 10^{-3}m = 1mm$

40. $\frac{I_1}{I_2} = \frac{100}{1}$
Now, $\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right]^2 = \left[\frac{\sqrt{100}+1}{\sqrt{100}-1}\right]^2 = \frac{121}{81} = \frac{3}{2}$

41. Using relation, $d\sin\theta = n\lambda$

$\Rightarrow \sin\theta = \frac{n\lambda}{d}$

For $n = 3, \sin\theta = \frac{3l}{d} = \frac{3\times589\times10^{-19}}{0.589}$

$= 3\times10^{-6}$

$or\ \theta = sin^{-1}(3\times10^{-6})$

42. 44. Self explanatory

45. $E = \frac{V}{d} = \frac{0.5}{5\times10^{-7}} = 10^6 Vm^{-1}$

46. Self explanatory

47. Current flow is possible and $i = \frac{V}{R} = \frac{3}{300} = 10^{-2}A$

48. 53. Self explanatory

54. $S = \frac{I_g G}{I - I_g}$

$$= \frac{10\times10^{-3}\times55}{1-\frac{1}{100}} = \frac{55}{99}$$

$= 0.555\Omega$

55. $\frac{r_e}{r_p} = \frac{m_e v_e}{q_e B}\times\frac{q_p B}{m_p v_p}$

Given: $m_e v_e = m_p v_p$

And $q_e = q_p$

$\therefore \frac{r_e}{r_p} = \frac{1}{1}$

56. $\oint \vec{B}.\vec{dl} = \mu_0 I$

$$= 4\pi\times10^{-7}\times\frac{1}{4\pi}$$

$= 10^{-7}$Weber/meter

57. $F = Bqv\sin 30^0$

$$= 0.2\times1.6\times10^{-19}\times10^7\times\frac{1}{2}$$

$= 1.6\times10^{-13}$N

58. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.1} = 20\pi$

$v_{max} = r\omega = 0.2\times20\pi = 4\pi$ms^{-1}

59. $l = \frac{\lambda}{4}, \Rightarrow \lambda = 4l$

$\therefore$ Frequency, $n = \frac{v}{\lambda}$

Or $264 = \frac{330}{4l}$

$\Rightarrow l = \frac{330}{264\times4} = \frac{330}{1056}$

$= 0.3125\ meter = 31.25cm$

60. From first law of thermodynamics

$dQ = dU + dw$

$\therefore\ 110 = dU + 40$

Or $dU = 110 - 40 = 70$J

CHEMISTRY

ANSWER KEYS

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40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**
4	3	1	1	2	1	3	3	3	3	1	1	1
53	**54**	**55**	**56**	**57**	**58**	**59**	**60**					
1	3	2	1	2	4	4	3					

HINTS & SOLUTIONS

1. Zn rad is placed in 20% NH_4Cl solution
4. 1 At.wt of bivalent metal required 2F = 2N electrons
6. Threshold energy = Activation energy + Internal energy
7. Increase in rate of reaction is maximum for the reaction having the maximum activation energy
8. order = $\frac{3}{2}-1=\frac{1}{2}$
9. $\log\frac{k_2}{k_1}=\frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$
14. $Cr_2O_7^{-2} \rightarrow 2Cr^{+3}+6e^{-}\quad E_{naHCO_3}=\frac{84}{1}$
15. Particle concentration is same
16. $\frac{P^0-P}{P_0}$ = mole fraction of solute
17. $\Delta T_b = K_b \times \frac{Wt.of\ glucose}{molecular\ weight\ of\ glucose}\times\frac{1000}{Wt.of\ solvent}$
18. Boiling point depends on number of solute particles hence highest in 0.1 M $FeCl_3$
19. $\Delta T_f = K_f.m$

    $m=\frac{\Delta T_f}{K_f}=\frac{0.0054}{1.8}$ = 0.003

    ∴ 0.001 molal compound produces 0.003 molal ions

    ∴ $[pt(NH_3)_4Cl_2]Cl_2 \rightarrow [Pt(NH_3)Cl_2]^{2+} + 2Cl^{-}$
20. Adsorption theory does not explain the acid-base catalysis
22. Ore has lower density in froth floatation process
23. Zone refining
25. $K_2Cr_2O7 + 4NaCl + 6H_2SO_4 \rightarrow 2KHSO_4 + 4NaHSO_4 + 2CrO_2Cl_2 + 3H_2O$ Chromyl chloride
26. This corresponds to 3 unpaired electrons (Cr^{3+})
28. Reactivity order of hydrogens is $3^O > 2^O > 1^O$
29. $C_2H_5-Cl \xrightarrow{KCN} C_2H_5-CH \xrightarrow{H_3O^+}$

    (A) Acid
30. Pyrene (CCl_4) is used as fire extinguisher
31. $CHCl_3$ is an anaesthetic agent
34. Ethyl alcohol on dehydration at 140^OC with con H_2SO_4 acids to give ethers

38. $CH_3COONa + NaOH \xrightarrow[(Soda\,lime)]{CaO} CH_4 + Na_2CO_3$

39. The abnormal trend of 3O amines is explained in terms of steric effect. Note basic order of amines on the basis of pKf, reported in Finar

$(CH_3)2NH > CH_3NH_2 > (CH_3)_3N > NH_3$

pK_b 3.23 3.32 4 2 4.73

$(C_2H_5)_2NH > (C_2H_5)_3N > C_2H_5NH_2 > NH_3$

pKb 3.07 3.13 3.37 4.73

40. Cellulose in most abundant in nature

42. 4.48 litre $CH_4 = \frac{4.48}{22.4} = \frac{1}{5}$ = 0.2 mol

= 16 x 0.2 = 3.2 g CH_4

= 0.2 x 6.02 x 10^{23}

= 1.2 x 10^{22} molecules

43. Outermost electron of Rb (At. no. 37) is 5s. Hence, its quantum numbers are 5, 0, 0 ± ½.

44. This is as per statement of ionization energy

45. K^+(Z = 19), Ca^{2+}(Z = 20), Sc^{3+}(Z = 21) and Cl^- (Z = 17), all have 18 electrons each and as such are isoelectronic.

47. $^{u}CO_2 = \left(\frac{3RT_1}{44}\right)^{1/2}$

$^{u}N_2 = \left(\frac{3R \times 294}{28}\right)^{1/2}$

$\left(\frac{3RT_1}{44}\right)^{1/2} = \left(\frac{3R \times 294}{28}\right)^{1/2}$

or T_1 = 462 K or 189^{O}C

48. T = 47 + 273 = 320 K

K.E. = $\frac{3}{2}$ RT (For 1 mole)

= $\frac{3}{2}$ x (8.314 JK^{-1} mol^{-1}) x (320 K)

= 3990.7 J mol^{-1}

Molar mass of O_2 = 32 g mol^{-1}

∴ K.E. (per gram of O_2) = $\frac{3990.7}{32}$ Jg^{-1}

= 124.7 Jg^{-1}

= 1.247 x 102 Jg^{-1}

49. $\Delta n_g = 4 - 2 = +2$

Thus ΔH > ΔE

50. As the equation is multiplied with 2, the equilibrium constant becomes square of the original value

51. Precipitation occurs only when ionic product exceeds solubility product.

In case (A) ionic product

= 10^{-4} x 10^{-4}

= 10^{-8} > 1.8 x 10^{-10}

53. $C_3C_7COONa + NaOH \xrightarrow{CaO} C_3H_8 + Na_2CO_3$

55. Most hazardous metal pollutant of automobile exhaust is lead

56. Ca^{2+}, K^+ and S^{2-} are isoelectronic species and for these greater the nuclear charge (atomic number), smaller will be the size.

57. Tranquilizers affect the CNS system and induce sleep

58. As atomic number increases Halogen gains electrons less readily
59. Catalytic poison for Fe is H_2S.
60. $\frac{m_1}{m_2}-\frac{E_1}{E_2}250\frac{m_1}{m_2}=\frac{E_1}{E_2}$

MATHEMATICS

ANSWER KEYS

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3	2	1	4	3	1	4	2	1	3	1	2	4
27	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
1	3	4	2	2	3	1	2	4	3	3	2	1
40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**
4	3	2	1	3	2	2	3	1	2	3	4	2
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1	4	3	1	2	4	3	2					

HINTS & SOLUTIONS

1. $y = \frac{2x+1}{3}$

   $\frac{3y-1}{2} = x$

   $\frac{3y-1}{2} = f^{-1}(y)$

   $\frac{3x-1}{2} = f^{-1}(x)$

2. $\frac{2^{x+3}}{2^{x-1}} = 2^4 = f(4)$

3. $f(x+y) = f(x)f(y)$

   $\Rightarrow f(x) = P^x$

   $f(5) = 32 = P^5$

   $\Rightarrow$ P = 2

   $f(7) = 2^7 = 128$

4. $25 - x^2 \geq 0$

   $x^2 \leq 25$

5. $\sin \cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)$

   $= \sqrt{\frac{x^2+1}{x^2+2}}$

6. $\frac{\pi}{4} = \tan^{-1} 1$

   $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

7. Standard result

8. Standard result

9. Tr (A–2B)

   = Tr(A) – 2 Tr(B)

10. Standard result

11. |Adj A| = $|A|^{n-1}$

13. $C_1 \rightarrow C_1 - C_2$

$(a + b)^2 + (a - b)^2 = 4ab$

14. Determinant of skew-symmetrix

Matrix of order 3 is 0

15. $\underset{x\to 1}{Lt} \frac{\log x}{x-1} = K$

$\underset{x\to 1}{Lt} \frac{\frac{1}{x}}{1} = K$ (L.H. Rule)

1 = K

16. $\underset{x\to 1}{Lt}\, x\sin\left(\frac{1}{x}\right) = 0 = f(0)$

$\therefore$ It is continuous

$f(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \sin\left(\frac{1}{h}\right)$ = does not exist

17. $h(x) = e^{\sin^{-1} x}$

$h'(x) = e\sin^{-1} x \times \frac{1}{\sqrt{1-x^2}}$

18. $x = \frac{\sin y}{\sin(a+y)}$

$\frac{dy}{dx} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$

$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

19. Take $x = \cos\theta$

$y = \frac{1}{2}(1-\cos\theta)$

$y = \frac{1}{2}(1-x)$

20. $g(x) = f^{-1}(x)$

$\Rightarrow f[f(x)] = x$

$\Rightarrow g'(f(x))\cdot f'(x) = 1$

$f(0) = 1 \quad f'(0) = \frac{1}{2}$

$g'(f(0)) = \frac{1}{f'(0)}$

$g'(1) = 2$

21. $\frac{dy}{dt} = 2\frac{dx}{dt}$

$y^2 = 18x$

$$2y\frac{dy}{dt} = 18\frac{dx}{dt}$$

$\Rightarrow\ y = \frac{9}{2}$ and $x = \frac{9}{8}$

22. $y = y_1 = -\frac{1}{m}(x - x_1)$

$$m = \frac{dy}{dx}(a, a)$$

23. $f(x) = 1 - \sin^2 x \cos^2 x$

$$f(x) = 1 - \frac{1}{2}\sin^2 2x$$

$$f'(x) = 0 \quad \Rightarrow \qquad x = \frac{\pi}{4}, \frac{\pi}{2}$$

$$f''\left(\frac{\pi}{4}\right) > 0$$

$$f''\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

24. $f'(x) = \frac{-(2+x)}{(1+x)^2} < 0$, for x > 0

25. By parts $\frac{d}{dx}\left(\log\tan\left(\frac{1}{2}\right)\right) = \frac{1}{\sin x}$

26. $\int x^{n-1}\left(\frac{1}{x^n} - \frac{1}{x^n + 1}\right)dx$

$$\int\left(\frac{x^{n-1}}{x^n} - \frac{x^{n-1}}{x^n + 1}\right)dx$$

$$\frac{1}{n}\left[\log x^n - \log(x^n + 1)\right] + c$$

27. $\cos x + \sqrt{3}\sin x$

$$= 2\left(\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3}\right)$$

$$= 2\cos\left(x - \frac{\pi}{3}\right)$$

$$\therefore \quad I = \frac{1}{2}\int \sec\left(x - \frac{\pi}{3}\right)dx$$

28. $\int \sec^2(xe^x)\cdot\frac{d}{dx}(xe^x)\,dx$

29. $\int_0^{\frac{\pi}{4}} \sin^n x = \int_0^{\frac{\pi}{4}} \cos^n x$

30. Standard result

31. $\log_e^x = t \Rightarrow e^t = x$

$$\Rightarrow\ e^t dt = dx$$

32. Area = $\int_{1}^{2}(x^2+2)\,dx$

33. $x^3=\sqrt{x}\Rightarrow x^6=x$

$\Rightarrow\ x=0\,or\,1$

Area = $\int_{0}^{1}\left(\sqrt{x}-x^3\right)dx$

34. Area = $\frac{(a)(a)}{3}=1$

35. $\left(\frac{d^2y}{dx^2}\right)^3=y^4$

36. $\int y\,dy=\int(c-x)\,dx+K$

$x^2+y^2-2cx=2K$

37. Divided by cos x

$\Rightarrow\ \frac{dy}{dx}+\tan x\,dx=\sec x$

$I.F=\int_{e}\tan x\,dx=e^{\log\sec x}=\sec x$

38. Common between A$\cup$B and A is A

39. $\tan\theta=\cot(90-\theta)$ and $\tan\theta\times\cot\theta=1$

40. P(4) : 2^4 < 1 x 2 x 3 x 4 is true

41. Standard result

42. ${}^{12}C_3-{}^{7}C_3$

43. Among 52 terms 26 terms are cancelled

44. A.M $\geq$ G.M

$\frac{3^x+3^{1-x}}{2}\geq\sqrt{3^x\times3^{1-x}}$

45. x + y = 5

3 + 2 = 5

46. $r=\sqrt{(4-1)^2+(6-2)^2}$

$r=5, A=\pi r^2$

47. $\lim_{x\to0}\frac{\frac{1}{\sin x}-\frac{\cos x}{\sin x}}{x}$

$\lim_{x\to0}\frac{1-\cos x}{x\sin x}$

$\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{x\times2\sin\frac{x}{2}\times\cos\frac{x}{2}}$

$$\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}\times 2}=\frac{1}{2}$$

48. Given $p\wedge q$

$N(p\wedge q) = Np\vee Nq$

49. Standard result

50. $\frac{1}{2}|\overrightarrow{AB}+\overrightarrow{AC}|$

51. $a\cdot(\vec{b}\times\vec{c})=(\vec{c}\times\vec{a})\cdot\vec{b}$

$\vec{b}\cdot(\vec{c}\times\vec{a})=(\vec{a}\times\vec{b})\cdot\vec{c}$

$\vec{c}\cdot(\vec{a}\times\vec{b})=(\vec{b}\times\vec{c})\cdot\vec{a}$

52. Substitute a point in options and verify

53. $1=\frac{a+1+2}{4}\Rightarrow a=1$

$2=\frac{2+b+1}{4}\Rightarrow b=5$

$-1=\frac{3+2+c}{4}=1c=-9$

$\sqrt{a^2+b^2+c^2}=\sqrt{81+25+1}=\sqrt{107}$

54. $\frac{h-x_1}{a}=\frac{-(ax_1+by_1+cz_1+d)}{a_2+b_2+c_2}=\frac{k-y_1}{b}=\frac{l-z_1}{c}$

(a, b, c) = (2, –1, 1)

$(x_1, y_1, z_1) = (1, 3, 4)$

Find h, k and l

55. Standard result

56. Standard result

57. $P(B\cap A')=P(B)-P(A\cap B)$

$=P(A\cup B)-P(A)$

$P\left(\frac{B}{A'}\right)=\frac{P(B\cap A')}{P(A')}$

58. P(red or black) = $\frac{3+7}{3+4+7}$

$=\frac{10}{14}=\frac{5}{7}$

59. $P(A\cap B)=P(A)P(B)$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A\cup B)=P(A)+P(B)-P(B)$

60. Selection of one winning horse is $\frac{1}{5}$