MATHEMATICS

1. If A and B are two given sets then $A \cap (A \cap B)^C$ is equal to
   A) A B) B C) ϕ D) $A \cap B^C$

2. Let R be a relation in the set N of natural numbers defined by the relation $nRm \Leftrightarrow n$ is a factor of m. Then the relation R is
   A) reflexive and symmetric only B) symmetric and transitive
   C) reflexive and transitive D) an equivalence relation

3. The domain of $\sqrt{x-1} + \sqrt{8-x}$ is
   A) [1, 8] B) (–8, 8) C) [1, 8) D) (1, 8)

4. If $f(x) = \frac{x-1}{x+1}$, $x \neq -1$, then $f^{-1}(x)$ is
   A) $\frac{x+1}{x-1}$ B) $\frac{1+x}{1-x}$ C) $\frac{2}{1+x}$ D) $\frac{1}{x-1}$

5. The function $f : R \to R$ defined by $f(x) = [x]$, $\forall x \in R$ where [x] denotes the greatest integer less than or equal to x, is
   A) one-one B) onto
   C) both one-one and onto D) neither one-one nor onto

6. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is
   A) at least 30 B) at the most 20
   C) exactly 25 D) none of these

7. The identity element for the binary operation $*$ defined on $Q \sim \{0\}$ as
   $a * b = \frac{ab}{2}$ $\forall a, b \in Q - \{0\}$ is
   A) 1 B) 0 C) 2 D) none of these

8. Set A has 3 elements and set B has 4 elements. Then the number of injective mappings that can be defined from A to B is
   A) 144 B) 12 C) 24 D) 64

9. If $\tan^2 \theta = 2 \tan^2 \phi + 1$, then $\cos 2\theta + \sin^2 \phi$ is equal to
   A) 1 B) 0 C) –1 D) 2

10. The value of $\cos\left[\tan^{-1}\left(\tan 15 \frac{\pi}{4}\right)\right]$ is
    A) $\frac{1}{\sqrt{2}}$ B) $-\frac{1}{\sqrt{2}}$ C) 1 D) none of these

11. The equation $k \sin x + \cos 2x = 2k - 7$ possesses a solution if
    A) $k > 6$ B) $2 \le k \le 6$ C) $k > 2$ D) none of these

12. The number of diagonals that can be formed by a polygon of 100 sides is
    A) 4950 B) 4850 C) 5000 D) 10000

13. If $\tan \alpha$, $\tan \beta$ are the roots of $x^2 - 3x - 1 = 0$ then the value of $\tan(\alpha + \beta)$ is
    A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) none of these

14. The first three terms in the expansion of $(1 + ax)^n$, $(n \neq 0)$ are 1, 6x, $16x^2$. Then the values of a and n are respectively
    A) 2 and 9 B) 3 and 2 C) $\frac{2}{3}$ and 9 D) $\frac{3}{2}$ and 6

15. If n arithmetic means are inserted between 2 and 38, then the sum of the resulting series is obtained as 200. Then, the value of n is
    A) 6 B) 8 C) 9 D) 10

16. The amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is

A) $2\frac{\pi}{5}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{15}$ D) $\frac{\pi}{10}$

17. The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

A) $x^2 + y^2 = 9a^2$ B) $x^2 + y^2 = 16a^2$

C) $x^2 + y^2 = 4a^2$ D) $x^2 + y^2 = a^2$

18. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. Height of a point on the parabola which is 2 m from the centre is

A) 2.6 m B) 3.6 cm C) 4.6 m D) none of these

19. An ellipse has a minor axis of length 6 and distance between its foci is 8. Its equation is

A) $\frac{x^2}{6} + \frac{y^2}{5} = 1$ B) $\frac{x^2}{6} + \frac{y^2}{9} = 1$ C) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ D) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

20. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, the probability that the number formed is 35 is

A) $\frac{1}{10}$ B) $\frac{1}{20}$ C) $\frac{1}{30}$ D) none of these

21. Three of the six vertices of a regular hexagon are chosen at random. What is the probability that the triangle with these vertices is equilateral?

A) $\frac{3}{10}$ B) $\frac{3}{20}$ C) $\frac{1}{20}$ D) $\frac{1}{10}$

22. In a school there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. The probability that a randomly chosen student studies in class XII given that the chosen student is a girl is

A) 0.2 B) 0.1 C) 0.4 D) none of these

23. If $2P(A) = P(B) = \frac{5}{13}$, and $P(A|B) = \frac{2}{5}$ then $P(A \cup B)$ is

A) $\frac{11}{12}$ B) $\frac{11}{36}$ C) $\frac{11}{26}$ D) $\frac{10}{26}$

24. Coefficients of variation of two distributions are 50 and 60, and their arithmetic means are 30 and 25 respectively. Difference of their standard deviation is

A) 0 B) 1 C) 1.5 D) 2.5

25. Which of the following is not a statement?

A) Smoking is injurious to health.

B) 2 + 2 = 4.

C) 2 is the only even prime number.

D) Come here.

26. If the line joining two points A(2, 0) and B(3, 1) is rotated about A in anticlockwise direction through an angle of 15°, the equation of the line in new position is

A) $y - \sqrt{3}x + 2\sqrt{3} = 0$ B) $x - \sqrt{3}y + 2\sqrt{3} = 0$

C) $x - \sqrt{3}y = 0$ D) $x + \sqrt{3}y - 2\sqrt{3} = 0$

27. The length of the perpendicular drawn from the point P(3, 4, 5) on y-axis is

A) 10 B) $\sqrt{34}$ C) $\sqrt{113}$ D) $5\sqrt{2}$

28. Equation of the plane passing through the point (5, 2, –4) and perpendicular to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-1}$ is

A) $x + 2y - z = 10$ B) $2x - y - z = 10$

C) $2x + 3y - z = 20$ D) $x - y + 2z = 20$

29. Distance between two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

A) 12 units B) 4 units C) 8 units D) $\frac{2}{\sqrt{29}}$ units

30. The unit vector perpendicular to both $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

A) $\hat{i} - \hat{j} + \hat{k}$ B) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ C) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ D) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

31. Area of the parallelogram whose diagonals are $\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} - 2\hat{j} + \hat{k}$ is

A) $\frac{1}{2}\sqrt{20}$ B) $\frac{1}{2}\sqrt{5}$ C) $\frac{1}{2}\sqrt{80}$ D) $\frac{1}{2}\sqrt{70}$

32. If $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then the angle between the two vectors $\vec{a}$ and $\vec{b}$ is

A) 0 B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$

33. If A and B are symmetric matrices of same order, then $(AB' - BA')$ is a

A) skew symmetric matrix B) null matrix
C) symmetric matrix D) none of these

34. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then $A + A' = I$ if the value of α is

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) π D) $\frac{3\pi}{2}$

35. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A) A B) $I - A$ C) I D) 3A

(where I denotes unit matrix of order of A)

36. If A is an invertible matrix, then which of the following is not true?

A) $A^{-1} = |A|^{-1}$ B) $(A^2)^{-1} = (A^{-1})^2$ C) $(A')^{-1} = (A^{-1})'$ D) none of these

37. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then the value of $|A|\ |\text{adj } A|$ is

A) a^3 B) a^6 C) a^9 D) a^{27}

38. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ where $0 \le \theta \le 2\pi$, then

A) Det A = 0 B) Det A = $(2, \infty)$ C) Det A $\in (2, 4)$ D) Det A $\in [2, 4]$

39. The lines $\frac{x+3}{-3} = \frac{y-1}{k} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar if k is equal to

A) -1 B) 1 C) 2 D) -2

40. Equation of the plane passing through the line of intersection of the two planes $x + y + z = 6$ and $2x + 3y + 4z = -5$ and passing through the point (1, 1, 1) is

A) $20x + 23y + 26z = 69$ B) $14x + 15y + 13z = 38$
C) $25x + 24y + 23z = 68$ D) none of the above

41. The function $f(x) = [x]$ where $[x]$ denotes the greatest integer function is continuous at

A) 4 B) -2 C) 1 D) 1.5

42. The value of k which makes the function defined by $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$, continuous at $x = 0$ is

A) 8 B) 1 C) -1 D) none of these

43. The function $f(x) = e^{|x|}$ is

A) continuous everywhere but not differentiable at $x = 0$
B) continuous and differentiable everywhere
C) not continuous at $x = 0$
D) none of these

44. The local minimum value of the function $f(x) = 3 + |x|, x \in R$ is

A) -1 B) 3 C) 1 D) 0

45. The volume of a ball is increasing at the rate of 4π c.c./sec. The rate of increase of the radius when the volume is 288π c.c. is

A) $\frac{1}{6}$ cm/sec B) $\frac{1}{36}$ cm/sec C) $\frac{1}{9}$ cm/sec D) $\frac{1}{24}$ cm/sec.

46. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of

A) $1\ m^3/h$ B) $0.1\ m^3/h$
C) $1.1\ m^3/h$ D) $0.5\ m^3/h$

47. If f and g are two differentiable functions satisfying $g'(a) = 2$, $g(a) = b$ and $fog = I$ (identity function) then $f'(b)$ is equal to

A) $\frac{1}{2}$ B) 2 C) $\frac{2}{3}$ D) none of these

48. A car travels a distance S mts in t seconds given by $S = 20t - 4t^2$. Before coming to rest, it moves a distance of

A) 25 m B) 20 m C) 40 m D) 30 m

49. If $y = \log\left(\frac{1 - \cos x}{1 + \cos x}\right)$, then $\frac{dy}{dx}$ is equal to

A) 2 cosec x B) 2 sec x C) 2 tan x D) 2 cot x

50. If $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$, then $\frac{dy}{dx}$ is equal to

A) 2 B) $\frac{1}{2}$ C) 2x D) $\frac{2}{x}$

51. If $\int \frac{2^x}{\sqrt{1 - 4^x}} = k \sin^{-1}(2^x)$ then k is equal to

A) a log 2 B) $\frac{1}{2}$ C) $\frac{1}{2}\log 2$ D) $\frac{1}{\log 2}$

52. $\int \frac{f(x)\,\phi'(x) - f'(x)\,\phi(x)}{f(x)\,\phi(x)} [\log \phi(x) - \log f(x)]\,dx$ is equal to

A) $\log \frac{\phi(x)}{f(x)} + C$ B) $\frac{1}{2}\left[\log \frac{\phi(x)}{f(x)}\right]^2 + C$
C) $\frac{\phi(x)}{f(x)} \log \frac{\phi(x)}{f(x)} + C$ D) none of these

53. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then the constants A, B are

A) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ B) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ C) 0 and $-\frac{4}{\pi}$ D) $\frac{4}{\pi}$ and 0

54. If $f(x) = \int_{-1}^{x} |t|\,dt$ then for any $x \geq 0$, f(x) is equal to

A) $\frac{1}{2}(1 - x^2)$ B) $1 - x^2$ C) $\frac{1}{2}(1 + x^2)$ D) $1 + x^2$

55. The area bounded by the curve $xy = 4$, the x-axis and the line $x = 1$, $x = 3$ is

A) 2 log 3 sq.units B) 4 log 3 sq.units

C) 3 log 3 sq.units D) none of these

56. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq.unit. Then a is equal to

A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{2}$ C) 1 D) $\frac{1}{3}$

57. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$ is

A) 2 and 3 B) 2 and 4 C) 3 and 4 D) 1 and 2

58. The integrating factor of the differential equation $\frac{dy}{dx}(x \log x) + y = 2 \log x$ is given by

A) e^x B) log x C) log (log x) D) x

59. The minimum value of the expression $3^x + 3^{1-x}$, $x \in R$ is

A) 0 B) $\frac{1}{3}$ C) 3 D) $2\sqrt{3}$

60. If $\lim_{x \to 0} \left(\sin mx \cot \frac{x}{\sqrt{3}}\right) = 2$, then m is equal to

A) $\frac{\sqrt{3}}{2}$ B) $\frac{2}{\sqrt{3}}$ C) $\sqrt{3}$ D) $\frac{2}{3}$

Answer Key

Q.No.	Answer
1.	D
4.	B
7.	C
10.	A
13.	C
16.	D
19.	D
22.	B
25.	D
28.	C
31.	C
34.	B
37.	C
40.	A
43.	A
46.	A
49.	A
52.	B
55.	B
58.	B

Q.No.	Answer
2.	C
5.	D
8.	C
11.	B
14.	C
17.	C
20.	B
23.	C
26.	A
29.	D
32.	D
35.	C
38.	D
41.	D
44.	B
47.	A
50.	B
53.	D
56.	A
59.	D

Q.No.	Answer
3.	A
6.	C
9.	B
12.	B
15.	B
18.	B
21.	D
24.	A
27.	B
30.	B
33.	A
36.	A
39.	B
42.	D
45.	B
48.	A
51.	D
54.	C
57.	B
60.	B

1) $A \cap (A \cap B)^C = A \cap (A^C \cup B^C) = (A \cap A^C) \cup (A \cap B^C)$
$= \phi \cup (A \cap B^C) = A \cap B^C$.

2) Let $n \in N$. $\therefore$ n is a factor of n i.e., nR_n
i.e., R is reflexive. As 3 is a factor of 6, $3R_6$.
But 6 is not a factor of 3. $\therefore$ R is not symmetric. Let aRb, bRc.
Then $b = ak_1$, $c = bk_2$ for $k_1, k_2 \in N$
i.e., $c = bk_2 = a(k_1k_2)$ i.e., c is a multiple of a. $\therefore$ R is transitive.

3) $\sqrt{x-1} \geq 0$ $\therefore$ $x - 1 \geq 0$ i.e., $x \geq 1$. Also $\sqrt{8-x} \geq 0$
$\Rightarrow 8 \geq x$ i.e., $1 \leq x \leq 8$. $\therefore$ $x \in [1, 8]$

4) Let $f(x) = y = \frac{x-1}{x+1}$ $\therefore$ $y(x+1) = x - 1$

$$yx + y = x - 1$$
$$y + 1 = x - yx$$
$$y + 1 = x(1 - y)$$
$$\therefore x = \frac{1+y}{1-y}$$
$$\text{i.e., } f^{-1}(y) = x = \frac{1+y}{1-y}$$
$$\therefore f^{-1}(x) = \frac{1+x}{1-x}.$$

5) f is not one-one since $f(1.2) = f(1.3) = 1$.
But $1.2 \neq 1.3$. Also it is not onto as the range of $f = I \neq R$.

6) Let the number of newspapers be x. If every student reads one newspaper the number of students would be 60x. Since every student reads 5 newspapers,
number of students $= \frac{x \times 60}{5}$

i.e., $\frac{60x}{5} = 300$ $\therefore$ $x = 25$

7) Let e be the identity element.
Then $a * e = a = e * a$

i.e., $\frac{ae}{2} = a \Rightarrow e = 2$.

8) The total number of injective mappings from set A to set B is $^4P_3 = 4! = 24$.

9) $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - (2\tan^2\phi + 1)}{1 + 2\tan^2\phi + 1} = \frac{-2\tan^2\phi}{2(1 + \tan^2\phi)}$

$= -\frac{\tan^2\phi}{\sec^2\phi} = -\sin^2\phi$.

$\therefore$ $\cos 2\theta + \sin^2\phi = 0$.

10) $\tan^{-1}(\tan x) = x$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\tan 15\frac{\pi}{4} = \tan\left(4\pi - \frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

$\therefore$ $\tan^{-1}\left(\tan\frac{15\pi}{4}\right) = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$

$\therefore$ $\cos\left[\tan^{-1}\left(\tan\frac{15\pi}{4}\right)\right] = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

11) $k\sin x + \cos 2x = 2k - 7$
$k\sin x + 1 - 2\sin^2 x = 2k - 7$
$2\sin^2 x - k\sin x + 2(k - 4) = 0$

$\therefore \quad \sin x = \dfrac{k \pm \sqrt{k^2 - 16k + 64}}{4} = \dfrac{k \pm (k-8)}{4} = \dfrac{1}{2}(k-4)\ ,\ 2.$

But $\sin x \neq 2$. $\quad \therefore \ \sin x = \dfrac{1}{2}(k-4)$

$$-1 \le \sin x \le 1 \Rightarrow -1 \le \frac{1}{2}(k-4) \le 1$$

i.e., $-2 \le k - 4 \le 2$

$\therefore \ 2 \le k \le 6.$

12) Number of vertices = 100. $\therefore$ Number of lines = ${}^{100}C_2 = \dfrac{100 \times 99}{1 \times 2} = 4950.$

Number of sides = 100. $\therefore$ No. of diagonals = 4950 – 100 = 4850.

13) $\tan\alpha$, $\tan\beta$ are the roots of $x^2 - 3x - 1 = 0$

$\therefore$ Sum of the roots is $\tan\alpha + \tan\beta = 3$, product of the roots is $\tan\alpha \tan\beta = -1$.

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{3}{1+1} = \frac{3}{2}.$$

14) The first three terms of $(1 + ax)^n$ are ${}^nC_0$, ${}^nC_1\, ax$, ${}^nC_2\,(ax)^2$

i.e., $1,\ nax,\ \dfrac{n(n-1)}{2}\, a^2x^2$

$\therefore \ na = 6$ i.e., $n^2a^2 = 36 \quad \ldots(1) \quad \dfrac{n(n-1)}{2}\, a^2 = 16 \quad \ldots.(2)$

Dividing (1) by (2) $\dfrac{n^2a^2}{\dfrac{n(n-1)}{2}a^2} = \dfrac{36}{16}$ i.e., $\dfrac{2n}{n-1} = \dfrac{36}{16}$

$\therefore \ n = 9$ i.e., $9a = 6$. $\therefore \ a = \dfrac{2}{3}$.

15) $2, a_1, a_2, a_3, \ldots, a_n, 38$ are in A.P.

$\therefore \ \dfrac{n+2}{2}(2 + 38) = 200$ i.e., $(n + 2)\,20 = 200 \quad \therefore \ n + 2 = 10$

i.e., $n = 8$.

16) $r\cos\theta = \sin\dfrac{\pi}{5}$ and $r\sin\theta = 1 - \cos\dfrac{\pi}{5}$

$$\therefore \ \tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{1 - \cos\frac{\pi}{5}}{\sin\frac{\pi}{5}} = \frac{2\sin^2\frac{\pi}{10}}{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}$$

$\tan\theta = \tan\dfrac{\pi}{10} \quad \therefore \ \theta = \dfrac{\pi}{10}.$

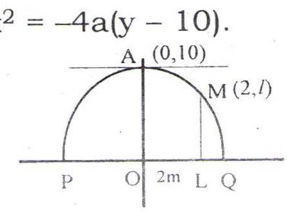
17) The centroid of the triangle coincides with the centre of the circle and radius of the circle is $\dfrac{2}{3}$ of the length of the median.

$\therefore \ r = \dfrac{2}{3} \times 3a = 2a$. Origin is the centre.

$\therefore$ Equation of the circle is $x^2 + y^2 = 4a^2$.

18) The vertex is A(0, 10). The equation of the parabola is $x^2 = -4a(y - 10)$.

$OQ = \dfrac{5}{2} \quad \therefore \ Q = \left(\dfrac{5}{2}, 0\right) \quad (\because \ PQ = 5\text{ m})$



Let M be $(2, l)$. The point Q lies on the parabola.

$\therefore \frac{25}{4} = -4a(0-10)$ i.e., $a = \frac{5}{32}$

M(2, l) lies on the parabola. $\therefore 4 = -4 \times \frac{5}{32}(l-10)$

Solving, $l = \frac{18}{5} = 3.6$.

19) $2b = 6 \therefore b = 3$. The distance between the foci is $2ae = 8$

$\therefore ae = 4$ i.e., $a^2e^2 = 16$ But $b^2 = a^2(1-e^2) = a^2 - a^2e^2$

$\therefore 9 = a^2 - 16$ i.e., $a^2 = 25 \therefore a = 5$.

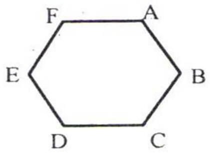
Equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

20) $n(S) = {}^5P_2 = 5 \times 4 = 20$

Let E be the event that the number formed is 35.

Then $n(E) = 1$. $\therefore$ Probability $= \frac{1}{20}$.

21) ABCDEF is a regular hexagon.



Total number of triangles is ${}^6C_3 = 20$.

(since no three points are collinear)

Of these only ΔACE and ΔBDF are equilateral.

$\therefore$ Required probability $= \frac{2}{20} = \frac{1}{10}$.

22) Let E denote the event that a student chosen randomly studies in class XII.

Let F be the event that the chosen student is a girl. Then $P(E|F) = \frac{P(E \cap F)}{P(F)}$.

$P(F) = \frac{430}{1000} = 0.43$, $P(E \cap F) = \frac{43}{1000}$ (10% of 430 is 43)

$= 0.043$.

$P(E|F) = \frac{0.043}{0.43} = 0.1$

23) $P(A) = \frac{5}{26}$, $P(B) = \frac{5}{13}$, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{2}{5}$ (given)

$\therefore \frac{2}{5} = \frac{P(A \cap B)}{\frac{5}{13}} \Rightarrow P(A \cap B) = \frac{2}{13}$

$\therefore P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$.

24) Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

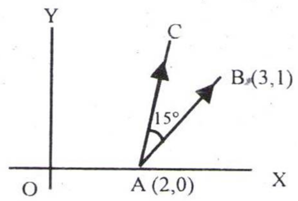
$\therefore \frac{\sigma_1}{\bar{x}_1} \times 100 = 50$, $\frac{\sigma_2}{\bar{x}_2} \times 100 = 60$

where $\bar{x}_1 = 30$, $\bar{x}_2 = 25$. $\therefore \frac{\sigma_1}{30} \times 100 = 50 \Rightarrow \sigma_1 = 15$

$\frac{\sigma_2}{25} \times 100 = 60 \Rightarrow \sigma_2 = 15 \quad \therefore \sigma_1 - \sigma_2 = 0$.

26) Slope of AB = $\frac{1-0}{3-2}$ = 1 = tan 45°

$\angle BAX$ = 45°. If AB is turned through an angle of 15°, then slope of AC is tan 60° = $\sqrt{3}$.



Equation of AC is $y - 0 = \sqrt{3}(x-2)$

i.e., $y = \sqrt{3}\,x - 2\sqrt{3}$ i.e., $y - \sqrt{3}\,x + 2\sqrt{3} = 0$.

27) Let Q be the foot of the perpendicular from P(3, 4, 5) on y-axis.

Then Q = (0, 4, 0).

$\therefore$ $PQ = \sqrt{(3-0)^2 + (4-4)^2 + (5-0)^2} = \sqrt{9+25} = \sqrt{34}$.

28) Required equation is $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$, where A, B, C are direction ratios of the normal line. (x_1, y_1, z_1) is the point which lies on the plane. Here A, B, C are 2, 3, –1 and $(x_1, y_1, z_1) = (5, 2, -4)$.

Required plane is $2(x-5) + 3(y-2) - 1(z+4) = 0$.

i.e., $2x + 3y - z = 20$.

29) From the equations of the two planes we see that they are parallel planes, since the coefficients of the second plane are two times that of the first plane. Choose a point on $2x + 3y + 4z = 4$ ...(1) say (0, 0, 1) (we do this by guess work).

This satisfies the equation (1).

The perpendicular from (0, 0, 1) on $4x + 6y + 8z - 12 = 0$ ...(2) is

$$\left|\frac{0+0+8-12}{\sqrt{16+36+64}}\right| = \frac{4}{\sqrt{116}} = \frac{4}{\sqrt{29\times 4}} = \frac{2}{\sqrt{29}}$$

30) The unit vector perpendicular to $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

$$\left|\frac{(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})}{(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})}\right| = \frac{\hat{k}-\hat{j}+\hat{i}}{\sqrt{1+1+1}} = \frac{\hat{k}-\hat{j}+\hat{i}}{\sqrt{3}}.$$

31) Required area = $\frac{1}{2}\,|\vec{a}\times\vec{b}|$ where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} - 2\hat{j} + \hat{k}$

$$\therefore\ \vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -1 & -2 & 1\end{vmatrix} = 8\hat{i} - 4\hat{j}$$

$$\frac{1}{2}\,|\vec{a}\times\vec{b}| = \frac{1}{2}\sqrt{64+16} = \frac{1}{2}\sqrt{80}.$$

32) $|\vec{a}\times\vec{b}| = \vec{a}\cdot\vec{b}$ $\therefore$ $|\vec{a}|\,|\vec{b}|\sin\theta = |\vec{a}|\,|\vec{b}|\cos\theta$

i.e., $\sin\theta = \cos\theta \Rightarrow \tan\theta = 1$ $\therefore$ $\theta = \frac{\pi}{4}$.

33) $(AB' - BA')' = (AB')' - (BA')' = BA' - AB'$

$= -(AB' - BA')$

$\therefore$ $AB' - BA'$ is a skew symmetric matrix.

34) $A = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ $A' = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$A + A' = \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}$. Given, $A + A' = I$.

$\begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ i.e., $2\cos\alpha = 1$ $\therefore$ $\cos\alpha = \frac{1}{2}$ i.e., $\alpha = \frac{\pi}{3}$.

35) $(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3IA^2 - 7A$

$= I + A^2\cdot A + 3A + 3A^2 - 7A$

Given $A^2 = A \quad \therefore I + A \cdot A + 3A + 3A - 7A$

$= I + A + 3A - 4A = I$

36) In answer (A), left hand side is a matrix and the right hand side is a determinant.

They cannot be equal.

37) $|A| = a^3$, $|\text{adj } A| = |A|^{n-1}$ if $|A| \neq 0$. Here n = 3

$\therefore \; |\text{adj } A| = |A|^2 = (a^3)^2 = a^6$

$|A| \cdot |\text{adj } A| = a^3 \cdot a^6 = a^9$.

38) Det A $= 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$= 2(1 + \sin^2\theta)$. As $\theta \in [0, 2\pi]$, $\sin\theta \in [-1, 1]$

$\therefore$ Det A $\in [2, 4]$.

39) Let $(x_1, y_1, z_1) = (-3, 1, 5)$, $(x_2, y_2, z_2) = (-1, 2, 5)$

$a_1, b_1, c_1 = -3, k, 5$, $a_2, b_2, c_2 = -1, 2, 5$

If two lines are coplanar then $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

i.e., $\begin{vmatrix} 2 & 1 & 0 \\ -3 & k & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0 \Rightarrow k = 1.$

40) Required plane is of the form $x + y + z - 6 + \lambda(2x + 3y + 4z + 5) = 0$

i.e., $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - 6 + 5\lambda = 0$

This passes through (1, 1, 1).

$\therefore \; 1 + 2\lambda + 1 + 3\lambda + 1 + 4\lambda - 6 + 5\lambda = 0$

i.e., $14\lambda = 3 \quad \therefore \lambda = \dfrac{3}{14}$

Required plane is

$$\left(1 + 2 \times \frac{3}{14}\right) x + \left(1 + 3 \times \frac{3}{14}\right) y + \left(1 + 4 \times \frac{3}{14}\right) z - 6 + 5 \times \frac{3}{14} = 0$$

i.e., $20x + 23y + 26z - 69 = 0$.

41) The greatest integer function [x] is discontinuous at all integral values of x.

42) $\lim\limits_{x \to 0} \sin\dfrac{1}{x}$ does not exist.

43) $f(x) = e^{|x|} = \begin{cases} e^x, & x \geq 0 \\ e^{-x}, & x < 0 \end{cases}$

f(x) is continuous everywhere as well as differentiable for each real $x \neq 0$.

At x = 0, left hand derivative $= \dfrac{d}{dx}(e^{-x})$ at x = 0

$= -e^{-x}$ at x

$= 0$ i.e., -1

Right hand derivative $= [e^x]_{x=0} = 1$

$\therefore$ f is not differentiable at x = 0.

44) We know $|x|$ is not differentiable at x = 0

i.e., 0 is a critical point of f. To the left of 0,

$f(x) = 3 - x. \quad \therefore f'(x) = -1 < 0$

To the right of 0, $f(x) = 3 + x \quad \therefore f'(x) = 1 > 0$.

$\therefore$ x = 0 is a point of local minima of f and local minimum value of f is f(0) = 3.

45) $V = \frac{4}{3}\pi r^3$. $\therefore \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$$\text{i.e., } 4\pi = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{1}{r^2}$$

Again $V = 288\pi$. $\therefore \frac{4}{3}\pi r^3 = 288\pi \Rightarrow r^3 = 72 \times 3 = 216$

i.e., $r^3 = 6^3 \quad \therefore \; r = 6$.

When $r = 6$, $\frac{dr}{dt} = \frac{1}{36}$ cm/sec.

46) $V = \pi r^2 h$ i.e., $V = 100\pi \times h$ $\quad \therefore \frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$$314 = 100\pi \times \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{314}{100\pi} = \frac{314}{100 \times 3.14} = \frac{3.14}{3.14} = 1$$

47) $fog = I \Rightarrow f[g(x)] = x \;\forall\; x$

$$\therefore f'[g(x)]\, g'(x) = 1$$

$$\therefore f'[g'(a)]\, g'(a) = 1$$

$$\therefore f'[g'(a)] = \frac{1}{g'(a)} = \frac{1}{2}$$

$$\text{i.e., } f'(b) = \frac{1}{2}.$$

48) $S = 20t - 4t^2 \quad \therefore \frac{dS}{dt} = \text{Velocity} = 20 - 8t$

When velocity = 0, the car stops, i.e., $20 - 8t = 0 \Rightarrow t = \frac{5}{2}$

Distance travelled by the car is $S = 20t - 4t^2$

$$= 20 \times \frac{5}{2} - 4 \times \frac{25}{4} = 25 \text{ m.}$$

49) $y = \log\left(\frac{1 - \cos x}{1 + \cos x}\right) = \log\left(\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) = \log\left(\tan^2\frac{x}{2}\right)$

$y = 2\log\left(\tan\frac{x}{2}\right) \qquad \therefore \frac{dy}{dx} = 2 \times \frac{1}{\tan\frac{x}{2}} \times \sec^2\frac{x}{2} \times \frac{1}{2}$

$$= \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \times \frac{1}{\cos^2\frac{x}{2}} = \frac{1}{\frac{1}{2}\sin x}$$

i.e., $\frac{dy}{dx} = 2 \operatorname{cosec} x$.

50) $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right) = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$$y = \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}.$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}.$$

51) Put $2^x = z$. Then $2^x \log 2\, dx = dz$.

$$\text{LHS} = \frac{1}{\log 2} \int \frac{dz}{\sqrt{1-z^2}} = \frac{1}{\log 2} \sin^{-1} z = \frac{1}{\log 2} \sin^{-1}(2^x)$$

$$\therefore\ k = \frac{1}{\log 2}.$$

52) Given $\int \frac{f(x)\,\phi'(x) - f'(x)\,\phi(x)}{f(x)\,\phi(x)} \log\left[\frac{\phi(x)}{f(x)}\right] dx$

$$= \int d\left[\frac{\phi(x)}{f(x)}\right] \times \frac{f(x)}{\phi(x)} \times \log\left[\frac{\phi(x)}{f(x)}\right] dx$$

$$= \int d\left[\frac{\phi(x)}{f(x)}\right] \times \frac{1}{\frac{\phi(x)}{f(x)}} \times \log\left[\frac{\phi(x)}{f(x)}\right] dx$$

$$= \int \log\left[\frac{\phi(x)}{f(x)}\right] \times d\left[\log \frac{\phi(x)}{f(x)}\right] dx$$

$$= \frac{1}{2}\left[\log \frac{\phi(x)}{f(x)}\right]^2 + C \quad [\because \text{ it is of the form } \int t\ dt \text{ where } t = \log\left[\frac{\phi(x)}{f(x)}\right]]$$

53) $\int_0^1 f(x)\,dx = \int_0^1 \left[A \sin\left(\frac{\pi x}{2}\right) + B\right] dx$

$$= \left[A - \frac{\cos\left(\frac{\pi x}{2}\right)}{\frac{\pi}{2}} + Bx\right]_0^1 = -\frac{2A}{\pi}\left[\cos\frac{\pi}{2} - \cos 0\right] + B$$

$$= \frac{2A}{\pi} + B = \frac{2A}{\pi} \text{ given.} \quad \therefore\ B = 0.$$

Consider $f'(x) = A \cdot \cos\left(\frac{\pi x}{2}\right)\frac{\pi}{2}$

$$f'\left(\frac{1}{2}\right) = \frac{\pi A}{2} \cdot \cos\left(\frac{\pi}{4}\right) = \frac{\pi A}{2\sqrt{2}} = \sqrt{2} \text{ given.}$$

$$\therefore\ A = \frac{4}{\pi}.$$

54) $f(x) = \int_{-1}^{x} |t|\,dt\ ,\ x \geq 0$

$$= \int_{-1}^{0} |t|\,dt + \int_{0}^{x} |t|\,dt = \int_{-1}^{0} -t\,dt + \int_{0}^{x} t\,dt$$

$$= -\left[\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_0^x = -\left(0 - \frac{1}{2}\right) + \left(\frac{x^2}{2} - 0\right)$$

$$= \frac{x^2}{2} + \frac{1}{2} = \frac{1 + x^2}{2}.$$

55) Required area $= \int_{x=1}^{3} y\,dx = \int_1^3 \frac{4}{x}\,dx = 4[\log x]_1^3 = 4(\log 3 - \log 1)$

$= 4 \log 3.$

56) We know that the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is

$$\frac{16a^2}{3} = \frac{(4a)\,(4a)}{3}$$

$\therefore$ Area enclosed by $y = ax^2$ i.e., $x^2 = \frac{y}{a}$ and $x = ay^2$

i.e., $y^2 = \frac{x}{a}$ is $\frac{\left(\frac{1}{a}\right)\left(\frac{1}{a}\right)}{3}$ i.e., $\frac{1}{3a^2} = 1$ (given).

$\therefore\ 3a^2 = 1 \quad \therefore\ a^2 = \frac{1}{3}$ i.e., $a = \frac{1}{\sqrt{3}}$.

57) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$

$\therefore\ \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}\times 12} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}\times 12}$

i.e., $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{9} = \left(\frac{d^2y}{dx^2}\right)^{4}$

$\therefore$ Order is 2, degree 4.

58) $\frac{dy}{dx}(x \log x) + y = 2 \log x$

$\therefore\ \frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$. Here $P = \frac{1}{x \log x}$, $Q = \frac{2}{x}$

I.F. $= e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$

59) We know Arithmetic mean ≥ Geometric mean for positive numbers.

$\therefore\ \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3^x \cdot 3^{1-x}}$

i.e., $\frac{3^x + 3^{1-x}}{2} \geq \sqrt{3^x \cdot \frac{3^1}{3^x}} = \sqrt{3}$.

i.e., $3^x + 3^{1-x} \geq 2\sqrt{3}$.

60) $\lim_{x\to 0}\left(\sin mx \cot \frac{x}{\sqrt{3}}\right) = 2$

$\lim_{x\to 0} \frac{\sin mx}{mx} \times mx \times \lim_{x\to 0} \frac{1 \times \frac{x}{\sqrt{3}}}{\tan \frac{x}{\sqrt{3}}} \times \frac{\sqrt{3}}{x} = 2$

i.e., $1 \times m \times 1 \times \sqrt{3} = 2. \quad \therefore\ m = \frac{2}{\sqrt{3}}$.