MODEL PAPER - III
MATHEMATICS

I PUC
Time: 3 Hrs **Max Marks: 100**

Part – A

I. Answer the following: **1 x 10 = 10**

1. **Ans:** $P(A)=2^4=16$
2. **Ans:** $n(A) = 2,\ n(B)=2$

   No of relation in $2^{n(A)n(B)}=2^4=16$
3. **Ans:** $\frac{5\pi^c}{3}=\frac{5\cancel{\pi}}{\cancel{3}}\times\frac{\overset{60}{\cancel{150}}}{\cancel{\pi}}=300^0$
4. **Ans:** Modulus = $\sqrt{1^2+1^2}=\sqrt{2}$
5. **Ans:** $\frac{8!}{6!\times2!}=\frac{{}^4\cancel{8}\times7\times\cancel{6!}}{\cancel{6!}\times\cancel{2}}=28$
6. **Ans:** $n = 9 + 8 = 17$
7. **Ans:** $(-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$

   $x^2+y^2-2x-2y=0$
8. **Ans:** $\lim\limits_{x\to0}\frac{\sin ax}{bx}=\frac{1}{b}\lim\limits_{x\to0}a\frac{\sin ax}{ax}$

   $=\frac{1}{b}\times a=\frac{a}{b}$
9. **Ans:** $\sqrt{2}$ is not a complex number
10. **Ans:** $n(s)=13$

    P(a Vowel in choosen) $=\frac{6}{13}$

Part – B

II. Answer any TEN of the following: **10 x 2 = 20**

11. **Ans:** $A=\{a,b\}\qquad B=\{1,2,3\}$

    $B\times A=\{(1,a),(2,a),(3,a),(1,b),(2,b),(3,b)\}$
12. **Ans:** $B\cap C=\{5,6\}$

    $A\cup(B\cap C)=\{1,2,3,4,5,6\}$
13. **Ans:** $(f+g)(x)=f(x)+g(x)=\sqrt{x}+x$

    $(fg)(x)=[f(x)][g(x)]=x\sqrt{x}$
14. **Ans:** In 60 minute, the minute hand of a clock completes one revolution

    $\therefore$ in 20 minutes, the minute hand turns through $\frac{1}{3}$ of revolution

    $\therefore\theta=\frac{1}{3}\times360^0=120^0\times\frac{\pi}{180}=\frac{2\pi}{3}^C$

    $\therefore S=r\theta$

    $S=2.1\left(\frac{2\pi}{3}\right)\qquad\because r=2.1cm$

    $S=\frac{(4.2)\pi}{3}=4.396$ cm

15. **Ans:** $\text{Sin}15^0 = \sin(45^0 - 30^0)$

$= \sin 45^0 \cos 30^0 - \cos 45^0 \sin 30^0$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

16. **Ans:** $Z = 1 + i\sqrt{3}$

$|Z| = r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$Z = r[\cos\theta + i\sin\theta]$

$Z = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

17. **Ans:** $\frac{5-2x}{3} \le \frac{x}{6} - 5$ multiply throughout by 18

$\overset{6}{\cancel{18}}\left(\frac{5-2x}{\cancel{3}}\right) \le 18\left(\frac{x}{6} - 5\right)$

$30 - 12x \le \frac{18x}{6} - 90$

$30 - 12x \le 3x - 90$

$30 - 12x - 3x \le \cancel{3x} - \cancel{3x} - 90$

$30 - 15x \le -90$

$\cancel{30} - \cancel{30} - 15x \le -90 - 30$

$-15x \le -120$

$x \ge 8$

$x \in (8, \infty)$

18. **Ans:** Slope of $x - 2y + 3 = 0$ is $\frac{1}{2}$

$\perp^r$ slope $= -2 = m$

w.k.t $y - y_1 = m(x - x_1)$

$y + 2 = -2(x\text{-}1)$

$2x + y = 0$

19. **Ans:** $A = (-2, 3, 5)$ $B = (1, 2, 3)$ $C = (7, 0, -1)$

$AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$

$BC = \sqrt{(7+1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = \sqrt{4\times 14} = 2\sqrt{14}$

$CA = \sqrt{(-2-7)^2 + (3-0)^2 + (5+1)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$

$\therefore CA = AB + BC$

$3\sqrt{4} = \sqrt{14} + 2\sqrt{14}$

$3\sqrt{14} = 3\sqrt{14}$

$\therefore$ A, B, C are collinear

20. **Ans:** w.k.t $d = \left|\frac{C_1 - C_2}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{-34-31}{\sqrt{15^2+8^2}}\right| = \left|\frac{-65}{\sqrt{225+64}}\right| = \frac{65}{\sqrt{289}}$

$= \frac{65}{17}$ units

21. **Ans:** $\lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\frac{\lim\limits_{x\to 1}\frac{x^{15}-1}{x-1}}{\lim\limits_{x\to 1}\frac{x^{10}-1}{x-1}}=\frac{15}{10}=\frac{3}{2}$

22. **Ans:** Converse: if a parallelogram is a rhombus then it in a square

Contrapositive : If a parallelogram is a not rhombus then it is a not a square

23. **Ans:** $C.V = 60\,;\sigma = 21\,;\bar{x} = ?$

$$C.V = \frac{\sigma}{\bar{x}}\times 100$$

$$6\cancel{0} = \frac{21}{\bar{x}}\times 10\cancel{0} \Rightarrow \bar{x} = \frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{6}}}\times 10$$

$$\bar{x} = \frac{70}{2} = 35$$

24. **Ans:** $n(s) = 8;\quad S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$E =$getting at-least two heads

$= \{HHH, HHT, HTH, THH\}\qquad n(E) = 4$

$$P(E) = \frac{4}{8} = \frac{1}{2}$$

Part – C

III. Answer any TEN of the following: 10 x 3 = 30

25. **Ans:** Let x be the set of students who like to play cricket

& y be the set of students who like to play Hockey

$n(x) = 30; n(y) = 18$

26. **Ans:** f(x) = ax + b

f(1) = 1 f(2) = 3

a + b = 1 (1) 2a + b = 3 (2)

(1) – (2)

$$\begin{array}{r} a+b=1 \\ 2a+b=3 \\ \underline{(-)\quad(-)\quad(-)} \\ -a=-2 \\ \boxed{a=2} \end{array}$$

Sub – a in (1)

b = 1 – a

b = 1 – 2 = -1 sub ‘b’ and a in f(x)

$\therefore f(x) = 2x - 1$

27. **Ans:** LHS = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x).\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$$=+\sin x\cos x\left[\tan x+\cot x\right]$$

$$=+\sin x\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]=+\cancel{\sin x\cos x}\left[\frac{\sin^2 x+\cos^2 x}{\cancel{\sin x\cos x}}\right]$$

$$=1\times1=1$$

28. **Ans:** $x+iy=\sqrt{\frac{a+ib}{c+id}}$

$$x-iy=\sqrt{\frac{a-ib}{c-id}}$$

$$(x+iy)(x-iy)=\sqrt{\frac{a+ib}{c+id}}\times\sqrt{\frac{a-ib}{c-id}}$$

$$x^2+y^2=\sqrt{\frac{a^2+b^2}{c^2+d^2}}$$

29. **Ans:** $Z=\frac{-16}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$$=\frac{-\overset{4}{\cancel{16}}(1-i\sqrt{3})}{\cancel{1+3}}=-4+i4\sqrt{3}$$

$$|Z|=\sqrt{16+48}=\sqrt{16(1+3)}=4\times2=8=r$$

$$\theta=\pi-\tan^{-1}\sqrt{3}=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

$$Z=r\left[\cos\theta+i\sin\theta\right]$$

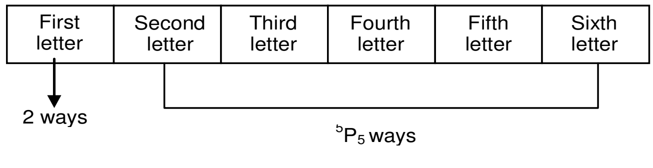
$$=8\left[\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right]$$

30. **Ans:** The word “MONDAY” consists of six different letters.

a) When only 4 letters are to be used, then the number of words formed = ${}^6P_6=\frac{6!}{2!}=6\times5\times4\times3=360$

b) When all the 6 letters are to be used, then the number of words formed = ${}^6P_6=6!=720$

c)



If the first letter is to be vowel, then it can b selected in two ways as there are only two vowels ‘A’ and ‘O’ corrsponding to each way of choosing the first letter. The remaining 5 letters can be arranged in 5 places is $^5P_5$ ways

Required number of words = $2\times{}^5P_5=2\times5!$

$$=2\times120=240$$

31. **Ans:** $(98)^5 = (100-2)^5$

$= {}^5C_0 100^5 + {}^5C_1(100)^4(-2) + {}^5C_2(100)^3(-2)^2 + {}^5C_3(100)^2(-2)^3 + {}^5C_4(100)(-2)^4 + {}^5C_5(100)^0(-2)^5$

${}^5C_0 = 1 = {}^5C_5$; ${}^5C_1 = 5 = {}^5C_4$; ${}^5C_2 = \frac{5!}{2!3!} = \frac{5 \times {}^2\cancel{4} \times {}^1\cancel{3!}}{\cancel{2} \times \cancel{3!}}$

${}^5C_2 = 10 = {}^5C_3$

$\therefore (98)^5 + 5(10^2)^4(-2) + 10(10^2)^3(4) + 10(10^2)^2(-8) + 5 \times 100(16) - 32$

= 10000000000 – 1000000000 + 40000000 – 800000 + 8000 – 32

= 9039207968

32. **Ans:** Let the three numbers in G.P be $\frac{a}{r}, a, ar$

Product = 1

$\left(\frac{a}{r}\right)a(ar) = 1$

$a^3 = 1$

$\Rightarrow \boxed{a = 1}$

Sum $= \frac{39}{10}$

$\frac{a}{r} + a + ar = \frac{39}{10}$

$a\frac{1}{r} + 1 + r = \frac{39}{10}$

$\frac{1 + r + r^2}{r} = \frac{39}{10}$

$10r^2 + 10r + 10 = 39r$

$10r^2 - 29r + 10 = 0$

$10r^2 - 25r - 4r + 10 = 0$

$5r(2r-5) - 2(2r-5) = 0$

$(5r-2)(2r-5) = 0$

$r = \frac{2}{5}$ or $r = \frac{5}{2}$

The three no's G.P are $\frac{a}{r}, a, ar$

$\therefore$ 3 no's are $\frac{5}{2}$, 1, $\frac{5}{5}$

33. **Ans:** Let x_1, x_2, x_3, x_4, x_5 be Five Arithmetic mean between 8 and 26.

$\therefore$ no's in A.P are 8, x, x_2, x_3, x_4, x_5, 26

$n = 7, a = 8, d = ?\ l = 26$

w.k.t

$l = a + (n-1)d$

$26 = 8 + 6d \Rightarrow 26 - 8 = 6d$

$18 = 6d \Rightarrow \boxed{d = 3}$

$x_1 = a + d = 8 + 3 = 11$

$x_2 = a + 2d = 8 + 6 = 14$

$x_3 = a + 3d = 8 + 9 = 17$

$x_4 = a + 4d = 8 + 12 = 20$

$x_5 = a + 5d = 8 + 15 = 23$

$\therefore$ The Five Arithmetic mean between 8 and 26 are 11, 14, 17, 20, 23

34. **Ans:** $a^2 = 16 \Rightarrow a = 4$

$b^2 = 9 \Rightarrow b = 3$

$c^2 = a^2 - b^2 = 7$

$c = \sqrt{7}$

Foci = $(\pm c, 0) = (\pm\sqrt{7}, 0)$

$$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$$

35. **Ans:** $f(x) = \cos x$

$f(x + \Delta x) = \cos(x + \Delta x)$

w.k.t $\frac{d}{dx} f(x) = \underset{\Delta x \to 0}{\text{lt}} \frac{f(x + \Delta x)}{\Delta x}$

$$\frac{d}{dx}\cos x = \underset{\Delta x \to 0}{\text{lt}} \frac{\cos(x + \Delta x) - \cos x}{\Delta x}$$

$$= \underset{\Delta x \to 0}{\text{lt}} \frac{-2\sin\left(\frac{x + \Delta x + x}{2}\right)\sin\left(\frac{x + \Delta x - x}{2}\right)}{\Delta x}$$

$$\underset{\Delta x \to 0}{\text{lt}} \frac{-\cancel{2}\sin\left(\frac{2x + \Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\cancel{2}\left(\frac{\Delta x}{2}\right)}$$

$$= -\sin\left(\frac{2x + 0}{2}\right) = -\sin x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

36. **Ans:** In this method, we assume that the given statement is false. That is we assume $\sqrt{5}$ is rational. This means that there exits positive integers a and b such that $\sqrt{5} = \frac{a}{b}$, where a and b have no common factors.

$5 = \frac{a^2}{b^2} \Rightarrow a^2 = 5b^2 \Rightarrow 5$ divides a

$\therefore$ there exist an integer c such that a = 5c

then $a^2 = 4c^2$ & $a^2 = 5b^2$

Hence $5b^2 = 4c^2 \Rightarrow b^2 = 5c^2 \Rightarrow 5$ divides b

But we have already shown that 5 division a. This implies that 5 in a common factor of both a and b which contradicts our earlier assumption that a and b have no common factors

This shows that the assumption $\sqrt{5}$ is irrational is wrong

Hence the statement $\sqrt{5}$ is irrational is true

37. **Ans:** When a card is drawn from a well shuffled seek of 52 cards, the number of possible outcomes in 52 n(s)=52

a) Let A be the event ‘the card drawn is a diamond’ clearly the number of elements in set A is 13 n(A)=13

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

b) We assume that the event ‘card drawn is an ace’ is B

$\therefore$ Card drawn is not an ace should be B’ $\therefore$ n(B)

$\therefore P(B') = 1 - P(B)$

$$= 1 - \frac{4}{52} = 1 - \frac{1}{13} = \frac{12}{13}$$

38. **Ans:** $$p(\text{not and not F}) = P(E' \cap F')$$

$$= P(E\cup F)' = 1 - P(E\cup F)$$

$$= 1-\left[P(E)+P(F)-P(E\cap F)\right]$$

$$= 1-\left\{\frac{1}{4}+\frac{1}{2}-\frac{1}{8}\right\} = \frac{8-(2+4-1)}{8}$$

$$= \frac{8-5}{8} = \frac{3}{8}$$

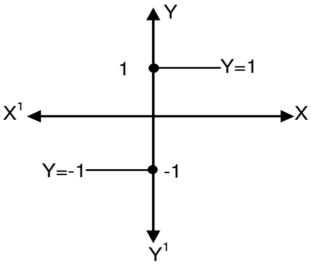
Part – D

IV. Answer any Six of the following: **6 x 5 = 30**

39. **Ans:** The function $f: R \to R$ defines by

$$f(x) = \begin{cases} 1, \text{if } x>0 \\ 0, \text{ if } x=0 \\ -1, \text{ if} x<0 \end{cases}$$

is called the signum function. The domain of the signum function is R and the range is the set {-1, 0, 1}



40. **Ans:** $$\text{LHS} = \cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)$$

$$= \frac{1+\cos 2x}{2} + \frac{1+\cos 2\left(x+\frac{\pi}{3}\right)}{2} + \frac{1+\cos 2\left(x-\frac{\pi}{3}\right)}{2}$$

$$= \frac{1}{2}\left[3+\cos 2x+\cos(2x+\frac{2\pi}{3})+\cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3+\cos 2x+2\cos\left(\frac{2x+\cancel{\frac{2\pi}{3}}+2x-\cancel{\frac{2\pi}{3}}}{2}\right).\frac{\cos(\cancel{2x}+\frac{2\pi}{3}-\cancel{2x}+\frac{2\pi}{3})}{2}\right]$$

$$= \frac{1}{2}\left[3+\cos 2x+2\cos 2x\cos\frac{4\pi}{3}\right]$$

$$= \frac{1}{2}\left[3+\cos 2x+2\cos 2x\cos\left(\pi+\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3+\cos 2x+2\cos 2x\left(-\cos\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3+\cos 2x+\cancel{2}\cos 2x\left(\frac{-1}{\cancel{2}}\right)\right]$$

$$= \frac{1}{2}\left[3+\cancel{\cos 2x}-\cancel{\cos 2x}\right] = \frac{3}{2}$$

41. **Ans:** $p(n):1^2+2^2+3^2+....+n^2=\frac{n(n+1)(2n+1)}{6}$

Let n = 1

$p(1):\text{LHS}=n^2=1^2=1$

$\text{RHS}=\frac{n(n+1)(2n+1)}{6}=\frac{1(1+1)(2+1)}{6}=\frac{2\times 3}{6}=1$

$\therefore \text{LHS}=\text{RHS}$

$p(n)$ is true for n = 1

Assume p(n) is true for n = m

$P(m)$: $1^2+2^2+3^2+....+m^2=\frac{m(m+1)(2m+1)}{6}$

Adding $(m+1)^{7h}$ term on both sides i.e, $(m+1)^2$

$$p(m+1):1^2+2^2+3^2+...+m^2+(m+1)^2=\frac{m(m+1)(2m+1)}{6}+(m+1)^2$$

$$=\frac{(m+1)\left[2m^2+m+6m+6\right]}{6}$$

$$=\frac{(m+1)\left[2m^2+7m+6\right]}{6}$$

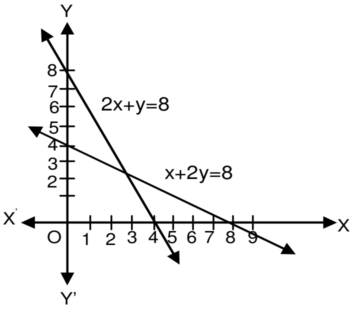
$$=\frac{(m+1)(m+2)(2m+3)}{6}$$

$$=\frac{(m+1)\left[(m+1)+1\right][2m+2+1]}{6}$$

$$=\frac{(m+1)\left[(m+1)+1\right]\left[2(m+1)+1\right]}{6}$$

$\therefore p(n)$ is true for n = m + 1

42. **Ans:** We draw the graphs of the lines x + 2y = 8 and 2x +y =8. Th inequality represents the region below the two lines, including the point on the respective lines
Since $x\geq 0, y\geq 0$, every point in the shaded region in the first quadrant represent a solution of the given system of inequalities.



43. **Ans:** 4 bowlers out of 5 can be selected in ${}^5C_4$ ways and 7 other players can be selected out of 17 - 5 = 12 in ${}^{12}C_7$

$\therefore$ Required number of ways = ${}^5C_4\times{}^{12}C_7$

$={}^5C_1\times{}^{12}C_5=5\frac{12!}{5!7!}$

$$=\frac{5\times{}^{1}\cancel{12}\times 11\times 10\times 9\times 8\times\cancel{7!}}{\cancel{5}\times\underset{1}{\cancel{4\times}}3\times\cancel{2}\times 1\times\cancel{7!}}$$

$=3960$

44. **Ans:** For any positive integer n

$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots + {}^nC_na^n$

Proof; The proof is by mathematical statement

$P(n):(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots. + {}^nC_na^n$

For n = 1

$P(1):(x+a)^1 = {}^1C_0x^1 + {}^1C_1x^0a = x+a$

Thus p(1) is true

Assume p(n) is true for n = m

$p(m):(x+a)^m = {}^mC_0x^m + {}^mC_1x^{m-1}a + {}^mC_2x^{m-2}a^2 + \ldots + {}^mC_ma^m$

We shall prove that p(m + 1) is also true i.e

$(x+a)^{m+1} = (x+a)(x+a)^m$

$= (x+a)^{m+1} = (x+a)(x+a)^m = (x+a)\{{}^mC_0x^m + {}^mC_1x^{m-1}a + {}^mC_2x^{m-2}a^2 + \ldots + {}^mC_na^m\}$

$= {}^mC_0x^{m+1} + {}^mC_1x^ma + {}^mC_2x^{m-1}a^2 + \ldots + {}^mC_ma^mx + {}^mC_0x^ma + {}^mC_1x^{m-1}a^2 + {}^mC_2x^{m-2}a^3 + \ldots + {}^mC_ma^{m+n}$

$= {}^mC_0x^{m+1} + ({}^mC_1 + {}^mC_0)x^ma + ({}^mC_2 + {}^mC_1)x^{m-1}a^2 + \ldots + ({}^mC_m + {}^mC_{m-1})xa^m + {}^mC_ma^{m+1}$

${}^mC_0 = {}^{m+1}C_0 = 1\,;\ {}^mC_m = {}^{m+1}C_{m+1} = 1$

w.r.t ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

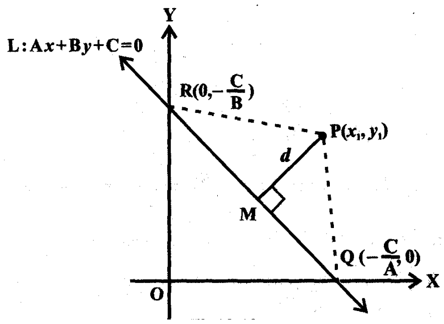
$\therefore {}^mC_1 + {}^mC_0 = {}^{m+1}C_1\,;{}^mC_2 + {}^mC_1 = {}^{m+1}C_2$

${}^mC_3 + {}^mC_2 = {}^{m+1}C_3 \ldots {}^mC_m + {}^{m+1}C_3 \ldots. {}^mC_m + {}^mC_{m-1} = {}^{m+1}C_m$

$(x+a)^{m+1} = {}^{m+1}C_0x^{m+1} + {}^{m+1}C_1x^ma + {}^{m+1}C_2x^{m-1}a^2 + \ldots + {}^{m+1}C_{m-1}xa^m + {}^{m+1}C_{m+1}a^{m+1}$

$\therefore p(n)$ is true for n = m + 1

45. **Ans:** The distance of a point from a line is the length of the perpendicular drawn from the point to the line: Let L : Ax + By + C = 0 be a line, whose distance from the point $P(x_1, y_1)$ is d. draw a perpendicular PM from the point P to the line L. if



The line meets the x and y- axes at the points Q and R, respectively. Then, coordinates of the points are $Q\left(-\frac{C}{A},0\right)$ and $R\left(0,-\frac{C}{B}\right)$. Thus, the area of the triangle PQR is given by

area $(\Delta PQR) = \frac{1}{2}PM.QR,$ which gives $PM = \frac{2\text{area}(\Delta PQR)}{QR}$ ....(1)

Also, area $(\Delta PQR) = \frac{1}{2}\left|x_1\left(o+\frac{C}{B}\right) + \left(-\frac{C}{A}\right)\left(-\frac{C}{B}-y_1\right) + o(y_1-o)\right|$

$= \frac{1}{2}\left|x_1\frac{C}{B} + y_1\frac{C}{A} + \frac{C^2}{AB}\right|$

or 2 area $(\Delta PQR) = \left|\frac{C}{AB}\right| . \left|A_{x_1} + B_{y_1} + C\right|$, and

$$QR=\sqrt{\left(0+\frac{C}{A}\right)^2+\left(\frac{C}{B}-0\right)^2}=\left|\frac{C}{AB}\right|\sqrt{A^2+B^2}$$

Substituting the values of area (ΔPQR) and QR in(1), weget

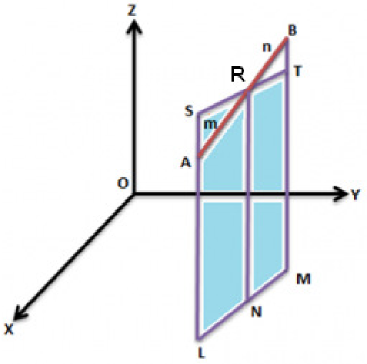
$$PM=\frac{\left|A_{x_1}+B_{y1}+C\right|}{\sqrt{A^2+B^2}}$$

Or $$d=\frac{\left|A_{x_1}+B_{y_1}+C\right|}{\sqrt{A^2+B^2}}$$

Thus, the perpendicular distance (d) of a line Ax + By +C = 0 from a point (x_1,y_1) is given by

$$d=\frac{\left|A_{x_1}+B_{y_1}+C\right|}{\sqrt{A^2+B^2}}$$

46. **Ans:** Let the two given points be $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$. Let the point R(x, y, z) divide PQ in the given ratio m : n internally. Draw PL, QM and RN perpendicular to the XY-plane. Obviously PL II RN II QM and feet of these perpendiculars lie in a XY-plane. The points L, M and N will lie on a line which is the intersection of the plane containing PL, RN and QM with the XY-plane. Through the point R draw a line ST parallel to the line LM. Line ST will intersect the line LP externally at the point S and the line MQ at T, as shown in Fig



Also note that quadrilaterals LNRS and NMTR are parallelograms.

The triangles PSR and QTR ar Similar. Therefore,

$$\frac{m}{n}=\frac{PR}{QR}=\frac{SP}{QT}=\frac{SL-PL}{QM-TM}=\frac{NR-PL}{QM-NR}=\frac{z-z_1}{z_2-z}$$

This implies $$z=\frac{mz_2+nz_1}{m+n}$$

Similarity, by drawing perpendiculars to the XZ and YZ –planes, we get

$$y=\frac{my_2+ny_1}{m+n}\text{and }x=\frac{mx_2+nx_1}{m+n}$$

Hence, the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n are

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m-n}\right)$$

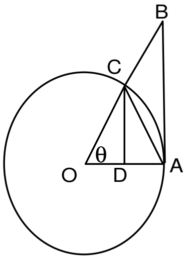
$$\text{Mid point }=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$=\left(\frac{1+5}{2},\frac{2+6}{2},\frac{3+7}{2}\right)=(3,4,5)$$

47. **Ans:** $\underset{\theta\to 0}{\text{lt}}\dfrac{\tan\theta}{\theta}=\underset{\theta\to 0}{\text{lt}}\dfrac{\sin\theta}{\theta\cos\theta}$

$=\dfrac{\underset{\theta\to 0}{\text{lt}}\dfrac{\sin\theta}{\theta}}{\underset{\theta\to 0}{\text{lt}}\cos\theta}=\dfrac{1}{1}=1$ Proof we know that $\sin(-\theta)=-\sin\theta$ and $\cos(-\theta)=\cos\theta$. Hence, it is sufficient to prove the inequality for $0<\theta<\dfrac{\pi}{2}$

In the fig O is the centre of the unit circle such that the angle AOC is θ radians and $0<\theta<\dfrac{\pi}{2}$. Line segments BA and CD are perpendiculars to OA. Further, join AC. Then



Area of ΔOAC < Area of sector OAC < Area of Δ OAB

i.e., $\dfrac{1}{2}\text{OA.CD}<\dfrac{\theta}{2\pi}.\pi(\text{OA})^2<\dfrac{1}{2}\text{QA.AB}.$

i.e., CD < x. OA < AB

From ΔOCD,

$\sin\ \theta=\dfrac{\text{CD}}{\text{OA}}(\text{since OC}=\text{OA})$ and hence $\text{CD}=\text{OA}\sin\theta.$ Also $\tan\theta=\dfrac{\text{AB}}{\text{OA}}$ and

Hence $\text{AB}=\text{OA}.\tan x,$ Thus $\text{OA}\sin\theta<\text{OA}\theta<\text{OA}\tan\theta$

Since length OA is positive, we have $\sin\theta<\theta<\tan\theta$

Since $0<\theta<\dfrac{\pi}{2}$, sin x is positive and thus by dividing throughout by sin x, we have $1<\dfrac{\theta}{\sin\theta}<\dfrac{1}{\cos\theta}$

Taking reciprocals throughout, we have

$$\cos\theta<\frac{\sin\theta}{\theta}<1$$

Taking limit as $\theta\to 0$ throughout we get $\underset{\theta\to 0}{\text{lt}}\cos\theta<\underset{\theta\to 0}{\text{lt}}\dfrac{\sin\theta}{\theta}<1$

$$1<\underset{\theta\to 0}{\text{lt}}\frac{\sin\theta}{\theta}=1$$

$\because$ there is no real number between 1 and 1

48. **Ans:**

x_i	f_i	f_ix_i	$\lvert x_i-\bar{x}\rvert$	$f\lvert x_1-\bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	N=25	$\sum f_ix_i=350$		$\sum f\lvert x_i-\bar{x}\rvert=158$

$$\bar{x}=\frac{\sum f_ix_i}{N}=\frac{350}{25}=15$$

$$\text{M.D}(\bar{x})=\frac{\sum f\lvert x_i-\bar{x}\rvert}{N}$$

$$=\frac{158}{25}$$

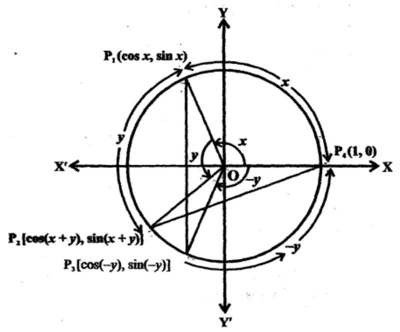
$$=6.32$$

Part – E

VI. Answer any One of the following: **1 x 10 = 10**

49. **Ans:** a) cos (x + y) = cos x cos y – sin x sin y

Consider the unit circle with centre at the origin. Let X be the angle P_4OP_1 and y be the angle P_1OP_2. Then (x + y) is the angle P_4OP_2 Also let (- y) be the angle P_4OP_3 Therefore, P_1. P_2. P_3 and P_4 will have the coordinates P_1(cos x, sin x), P_2[cos(x + y), sin (x + y)], P_3[cos (- y), sin(- y)1 and P_4(1, 0)



Consider the triangles P_1OP_3 and P_2OP_4. They are congruent (Why?). Therefore, P_1P_3 and P_2P_4 are equal. By using distance formula, we get

$$P_1P_3^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(y)]^2$$

$$= (\cos x - \cos y)^2 + (\sin x + \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y + 2\sin x \sin y$$

$$= 2 - 2(\cos x \cos y - \sin x \sin y) \qquad (\text{Why ?})$$

Also, $P_2P_4^2 = [1 - \cos(+y)]^2 + [0 - \sin(x+y)]^2$

$$= 1 - 2\cos(x+y) + \cos^2(x+y) + \sin^2(x+y)$$

$$= 2 - 2\cos(x+y)$$

Since $P_1P_3 = P_2P_4$, we have $P_1P_3^2 = P_2P_4^2$

Therefore, 2 -2 (cos x cos y – sin x sin y) = 2-2 cos(x + y)

Hence cos(x + y) = cos x cos y +sin x, sin y

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

Put x = y

$$\cos 2x = \cos x \cos x - \sin x \sin x$$

$$= \cos^2 x - \sin^2 x$$

b) Given series is $3.1^2 + 5.2^2 + 7.3^2 + \ldots.$

Let T_n denote the n^{th} term, then

$$T_n = (2n+1)n^2 = 2n^3 + n^2$$

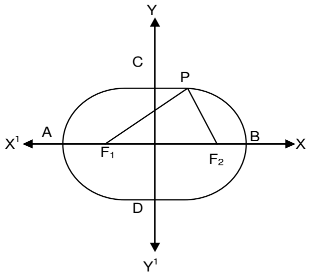
$$S_n = 2\sum n^3 + \sum n^2$$

$$= \cancel{2}\,\frac{n^2(n+1)^2}{\cancel{4}_{2}} + \frac{n(n+1)(2n+1)}{6}$$

$$\frac{n(n+1)}{2}\left\{n(n+1) + \frac{2n+1}{3}\right\}$$

$$= \frac{n(n+1)(3n^2+5n+1)}{6}$$

50. **Ans:** a) An ellipse is the set of all point in a plane, the sum of whose distances from two fixed points in the plane is a constant.



Let F_1 and F_2 be the foci and O be the midpoint of the line segment F_1F_2. Let O be the origin and the line from O through F_2 be the positive x- axis and that through F_1 be the negative x-axis. let the line through O perpendicular to the x-axis be the y-axis. Let the co-ordinate of F_1 be (-c,o) and F2 be (c, 0)

Let p(x, y) be any point on the ellipse such that the sum of the distance from P to the two foci be 2a

$$PF_1 + PF_2 = 2a$$

$$\sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$$

$$(x+c)^2 + \cancel{y^2} = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + (x-c)^2 + \cancel{y^2} \text{(squary on both sides)}$$

$$\cancel{x^2} + 2cx + \cancel{c^2} = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + \cancel{x^2} - 2cx + \cancel{c^2}$$

$$\cancel{4}cx = \cancel{4}a\left[a - \sqrt{(x-c)^2+y^2}\right]$$

$$cx = a^2 - a\sqrt{(x-c)^2+y^2} \quad (cx - a^2)^2 = [a\sqrt{(x-c)^2+y^2}]^2 \text{ squary on both sides}$$

$$c^2x^2 - 2cxa^2 + a^4 = a^2\left[(x-c)^2 + y^2\right]$$

$$c^2x^2 - 2\cancel{cxa^2} + a^4 = a^2x^2 - 2\cancel{cxa^2} + a^2c^2 + a^2y^2$$

$$a^4 - a^2c^2 = a^2x^2 - c^2x^2 + a^2y^2$$

$$a^2(a^2 - c^2) = x^2(a^2 - c^2) + a^2y^2 \div a^2(a^2 - c^2)$$

$$1 = \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ where } b^2 = a^2 - c^2$$

b) $f(x) = \dfrac{x + \cos x}{\tan x}$

$$f'(x) = \frac{\tan x(1 - \sin x) - (x + \cos x)\sec^2 x}{\tan^2 x}$$

$$= \frac{\tan x - \sin x \tan x - x\sec^2 x - \sec x}{\tan^2 x}$$