MODEL PAPER - II
MATHEMATICS

I PUC

Time: 3 Hrs **Max Marks: 100**

Part – A

I. Answer the following: **1 x 10 = 10**

1. **Ans:** The collection of all subsets of a set A is called the power set of A
2. **Ans:** $x+1=3 \Rightarrow x=2$
3. **Ans:** $\frac{7\pi^c}{6} = \left(\frac{7\cancel{\pi}}{\cancel{6}} \times \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}}\right)^0 = 210^0$
4. **Ans:** multiplicative inverse of $\sqrt{5}+3i$ is $\frac{1}{\sqrt{5}+3i} = \frac{\sqrt{5}-3i}{5+9} = \frac{-\sqrt{5}}{14} + \frac{i3}{14}$
5. **Ans:** $7!-5! = 7\times6\times5!-5! = 5!(7\times6-1) = 5!(41)$

   $=120\times41=4920$
6. **Ans:** $a_n = 4n-3$; $a_{17} = 68-3 = 65$
7. **Ans:** $6x+3y-5=0$

   $3y = -6x+5$

   $y = -2x+\frac{5}{3}$
8. **Ans:** $\underset{x\to0}{\text{lt}} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$
9. **Ans:** Intersection of two disjoint sets is an empty set
10. **Ans:** $P(A) = \frac{2}{11}$; $P(A') = 1-P(A)$

    $P(A') = 1-\frac{2}{11} = \frac{11-2}{11} = \frac{9}{11}$

Part – B

II. Answer any TEN of the following: **10 x 2 = 20**

11. **Ans:** $n(X\cap Y)=10$; $n(x)=28$; $n(y)=32; n(x\cup y)=?$

    $n(x\cup y) = n(x)+n(y)-n(x\cap y)$

    $= 28 + 32 - 10 = 50$
12. **Ans:** $A\times A\times A = \{(1,1,1),(1,1,-1),(1,-1,1),(1,-1,-1),(-1,1,1),(-1,1,-1)\ (-1,-1,1)(-1,-1,-1)\}$
13. **Ans:** $R = \{(x,y)\,|\,3x-y=0\ ,\ x,y\in A\}$

    $R = \{(1,3),(2,6),(3,9),(4,12)\}$

    Domain = $\{1,2,3,4\}$

    Range = $\{3,6,9,12\}$
14. **Ans:** $LHS = \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

    $= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 1 = \frac{1}{4}+\frac{1}{4}-1$

    $= \frac{1}{2}-1 = -\frac{1}{2}$

15. **Ans:** LHS = $\sin 2x = \dfrac{2\sin x\cos x}{1}$

$= \dfrac{2\sin x\cos x}{\cos^2 + \sin^2 x}$

$= \dfrac{\dfrac{2\sin x \cancel{\cos x}}{\cos^2 x}}{\dfrac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} + \dfrac{\sin^2 x}{\cos^2 x}}$ Divide each term of numerator and denominator by $\cos^2 x$

$= \dfrac{2\tan x}{1+\tan^2 x}$

16. **Ans:** $4x + 3 < 6x + 7$

$\cancel{4x} - \cancel{4x} + 3 - 7 < 6x - 4x + \cancel{7} - \cancel{7}$

$-4 < 2x$

$\dfrac{-4}{2} < \dfrac{\cancel{2}x}{\cancel{2}}$

$x > -2$

$x \in (-2, \infty)$

17. **Ans:** Let a be x- intercept and b be y – intercept

w.k.t Equation of line in intercept form is

$\dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow x + y = a \qquad (1)$

(1) passes through (2, 3)

$2 + 3 = a \Rightarrow a = 5$

Sub a in (1) $\qquad x + y = 5$

18. **Ans:** Slope of (h, 3) and (4, 1) is $\dfrac{1-3}{4-h} = \dfrac{-2}{4-h} = m_1$

Slope of line $7x - 9y - 19 = 0$ is $\dfrac{7}{9} = m_2$

At right angle

$m_1 m_2 = -1$

$\left(\dfrac{\cancel{-}2}{4-h}\right)\left(\dfrac{7}{9}\right) = \cancel{-}1$

$14 = 36 - 9h$

$9h = 36 - 14$

$9h = 22$

$h = \dfrac{22}{9}$

19. **Ans:** G = (1, 1, 1) A = (3, -5, 7) B = (-1, 7, -6) C= (x, y, z)

$G = \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}, \dfrac{z_1 + z_2 + z_3}{3}\right)$

$1 = \dfrac{3 - 1 + x}{3}$; $1 = \dfrac{-5 + 7 + y}{3}$; $1 = \dfrac{7 - 6 + z}{3}$

$3 = 3 - 1 + x \qquad 3 = 2 + y \qquad 3 - 1 = z$

x = 1 $\qquad$ y = 1 $\qquad$ z = 2

$\therefore C = (1, 1, 2)$

20. **Ans:** $\lim_{x\to 2} \frac{x^3-2x^2}{x^2-5x+6} = \lim_{x\to 2} \frac{x^2\cancel{(x-2)}^1}{\cancel{(x-2)}(x-3)} = \frac{2^2}{2-3} = -4$

21. **Ans:** Converse: If x is odd number then x is prime.
Contrapositive: If x is not odd number then x is not prime

22. **Ans:** 3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21
Median = 9

23. **Ans:** A and B are mutually exclusive events
$\therefore A\cap B=\phi$
$\therefore P(A\cap B)=0$
$P(A \text{ or } B)=P(A)+P(B)$
$=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

Part – C

III. Answer any TEN of the following: 10 x 3 = 30

24. **Ans:** Let U denote the set of group of students in a school,
T denotes the set of students taking Tea
C denotes the set of students taking coffee
$n(U)=600,\ n(T)=150,\ n(C)=225$
$n(T\cap C)=100\quad n(T'\cap C')=?$
$n(T'\cap C')=n((T\cup C)')$
$=n(U)-n(T\cup C)$
$=600-[n(T)+n(C)-n(T\cap C)]$
$=600-[150+225-100]$
$=600-275=325$
325 students were taking neither tea nor coffee

25. **Ans:** $(f+g)(x)=f(x)+g(x)=x+1+2x-3$
$=3x-2$
$(f-g)(x)=f(x)-g(x)=x+1-(2x-3)=x+1-2x+3$
$=-x+4$
$(fg)(x)=[f(x)][g(x)]=(x+1)(2x-3)$
$=2x^2-3x+2x-3=2x^2-x-3$

26. **Ans:** $\sin 2x+\cos x=0$
$2\sin x\cos x+\cos x=0$
$\cos x(2\sin x+1)=0$
$\cos x=0 \qquad 2\sin x=-1$
$x=n\pi, n\in I \qquad \sin x=-1/2$
$n\pi+(-1)^n\left(\frac{-\pi}{6}\right), n\in I$

27. **Ans:** $Z=1+i\sqrt{3}$
$|Z|=r=\sqrt{1+3}=2$
$\theta=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$
$Z=r[\cos\theta+i\sin\theta]=2\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]$

28. **Ans:** $x+iy=\frac{a+ib}{b+ia}$

$x-iy=\frac{a-ib}{b-ia}$

$(x+iy)(x-iy)=\frac{a+ib}{b+ia}\times\frac{a-ib}{b-ia}$

$x^2+y^2=\frac{a^2+b^2}{a^2+b^2}=1$

$x^2+y^2=1$

29. **Ans:** $5.{}^4P_r=6.{}^5P_{r-1}$

$\frac{5\times 4!}{(4-r)!}=\frac{6\times 5!}{[5-(r-1)]!}$

$\frac{{}^1\cancel{5}\times \cancel{4!}}{(4-r)!}=\frac{6\times \cancel{5}^1\times \cancel{4!}}{(6-r)!}$

$\frac{1}{\cancel{(4-r)!}}=\frac{6}{(6-r)(5-r)\cancel{(4-r)!}}$

$(6-r)(5-r)=6$

$(6-r)(5-r)=2\times 3$

$30-6r-5r+r^2=6$

$r^2-11r+24=0$

$r^2-8r-3r+24=0$

$r(r-8)-3(r-8)=0$

$r=3\quad \text{or}\quad r=8$

30. **Ans:** $x=3\ ;\ a=\frac{-x^3}{6},n=7$

n = 7 in odd therefore there is two middle terms i.e. $\frac{n+1}{2}=4^{th}$ term & $\frac{n+3}{2}=5^{th}$ term

4^{th} term

$r+1=4$

r = 3

$T_{r+1}={}^nC_r x^{n-r}a^r$

$T_4={}^7C_4 3^4\left(\frac{-x^3}{6}\right)^3$

$=-{}^7C_4\quad 3^4\frac{x^9}{6^3}=$

$=-\frac{-7!}{4!3!}\times\frac{3\times\cancel{3}\times\cancel{3}x^9}{\cancel{6}_2\times\cancel{6}_2\times\cancel{6}_2}\times\cancel{3}$

$=-\frac{-7\times\cancel{6}\times 5\times\cancel{4}}{\cancel{6}\times\cancel{4!}\times 8}x^3x^9=\frac{-105}{8}x^9$

5^{th} term

r +1 = 5

r = 4

$T_{r+1}={}^nC_r x^{n-r}a^r$

$T_5={}^7C_r 3^3\left(\frac{-x^3}{6}\right)^4$

$={}^7C_5\frac{27\times x^{12}}{6^4}$

$=\frac{27}{64}\ {}^7C_5 x^{12}$

31. **Ans:** $S_n = 116, a = 25 \quad d = -3$

W.K.T $\qquad n = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$116 = \frac{n}{2}[50 + (n-1)(-3)]$$

$$232 = n[50 - 3n + 3]$$

$$232 = n[53 - 3n]$$

$$3n^2 - 53n + 232 = 0$$

$$3n^2 - 24n - 29n + 232 = 0$$

$$3n(n-8) - 29(n-8) = 0$$

$n = 8$ or $n = \frac{29}{3}$ Neglect $\because n$ cannot be fraction $\therefore n = 8$

$$T_n = a + (n-1)d$$

$$T_8 = 25 + (8-1)(-3) = 25 - 21 = 4$$

32. **Ans:** We have $T_m = a + (m-1)d = n \qquad (1)$

$T_n = a + (n-1)d = m \qquad (2)$

Solving (1) and (2) we get

$(m-n)d = n - m$ or $d = -1$

a = n + m – 1

$$T_p = a + (p-1)d$$

$$T_p = n + m - 1 + (p-1)(-1)$$

$$T_p = n + m - p$$

33. **Ans:** $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a = 3,\ b = 4\ ; c = \sqrt{a^2 + b^2} = 5$

$e = \frac{c}{a} = \frac{5}{3};\ \text{foci}(\pm c, 0) = (\pm 5, 0)$

LLR= $\frac{2b^2}{a} = \frac{2 \times 16}{3} = \frac{32}{3}$

34. **Ans:** $f(x) = \sin x$

$$f(x + \Delta x) = \sin(x + \Delta x)$$

w.k.t $\frac{d}{dx}(f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$\frac{d}{dx}(\sin x) = \lim_{\Delta x \to 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\cos\left(\frac{x + \Delta x + x}{2}\right)\sin\left(\frac{\cancel{x} + \Delta x - \cancel{x}}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{{}^{1}\cancel{2}\cos\frac{(2x + \Delta x)}{2}\sin\frac{\Delta x}{2}}{{}_{1}\cancel{2}\left(\frac{\Delta x}{2}\right)} = \cos\left(\frac{2x + 0}{2}\right) \times 1$$

= cos x

35. **Ans:** In this method, we assume that the given statement in false. That is we assume that $\sqrt{7}$ is rational. This means that there exists positive integers a and b such that $\sqrt{7} = \frac{a}{b}$, where a and b have no common factors. Squaring the equation we get $7 = \frac{a^2}{b^2} \Rightarrow a^2 = 7b^2 \Rightarrow 7$ divides a. Therefore, there exists an integer c such that a = 7 c then $a^2 = 49c^2$ & $a^2 = 7b^2$ Hence $7b^2 = 49c^2 \Rightarrow b^2 = 7c^2 \Rightarrow 7$ divides b But we have already shown that 7 divides a. This implies that 7 is a common factor of both a and b which contradicts our earlier assumption that a and b have no common factors. This shown that the assumption $\sqrt{7}$ is irrational is true

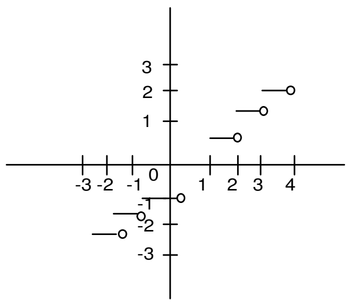
36. **Ans:** There are 9 discs in all so the total number of possible outcomes in 9. Let the events A, B, C be defines as
A : the disc drawn in red
B: the disc drawn in yellow
C: the disc drawn is blue
i) The number of red discs = 4 i.e,.n(A) = 4
$P(A) = \frac{4}{9}$
ii) The number of yellow discs = 2 i.e., n(B) = 2 $P(B)=\frac{2}{9}$
iii) The number of blue discs = 3 i.e. n(C) = 3 $P(C)=\frac{1}{3}$

37. **Ans:** i) $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$
$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$
ii) $P(\text{not } E \text{ and not } F) = P(E' \cap F') = P((E \cup F)')$
$= 1 - P(E \cup F)$
$= 1 - P(E) - P(F) + P(E \cap F)$
$= 1 - \frac{1}{4} - \frac{1}{2} + \frac{1}{8} = \frac{8-2-4+1}{8}$
$= \frac{3}{8}$

Part – D

IV. Answer any Six of the following: 6 x 5 = 30

38. **Ans:** The function $f : R \rightarrow R$ defined by $f(x) = [x], x \in R$ assume the value of the greatest integer, less than or equal to x. such a function is called the greatest integer function.
Domain = R (set of real number)
Range = set of integers



39. **Ans:** LHS = $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\frac{\cos 4x+\cos 2x+\cos 3x}{\sin 4x+\sin 2x+\sin 3x}$

$$=\frac{2\cos(\frac{4x+2x}{2})\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x\,\cancel{(2\cos x+1)}}{\sin 3x\,\cancel{(2\cos x+1)}}=\cot 3x$$

40. **Ans:** $P(n):1^2+3^2+5^2+....+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$

For n = 1

$P(n):\text{LHS}=(2n-1)^2=(2-1)^2=1^2=1$

$\text{RHS}=\frac{n(2n-1)(2n+1)}{3}=\frac{1(2-1)(\cancel{2+1})}{\cancel{3}}=1$

$\therefore$ LHS = RHS

$\therefore P(n)$ is true for n = 1

Assume P(n) is true for n = m

$P(m):1^2+3^2+5^5+....+(2m^{-1})^2=\frac{m(2m-1)(2m+1)}{3}$

We shall now prove that P(m + 1)is also true

By Adding $(2m + 1)^2$ on both sides

$$P(m+1):1^2+3^2+5^2+...+(2m-1)^2+(2m+1)^2=\frac{m(2m-1)(2m+1)}{3}+(2m+1)^2$$

$$=(2m+1)\left[\frac{m(2m-1)}{3}+2m+1\right]$$

$$=(2m+1)\left[\frac{2m^2-m+6m+3}{3}\right]$$

$$=(2m+1)\left[\frac{2m^2+5m+3}{3}\right]$$

$$P(m+1):1^2+3^2+5^2+...+(2m-1)^2+(2m+1)^2=\frac{(2m+1)(m+1)(2m+3)}{3}$$

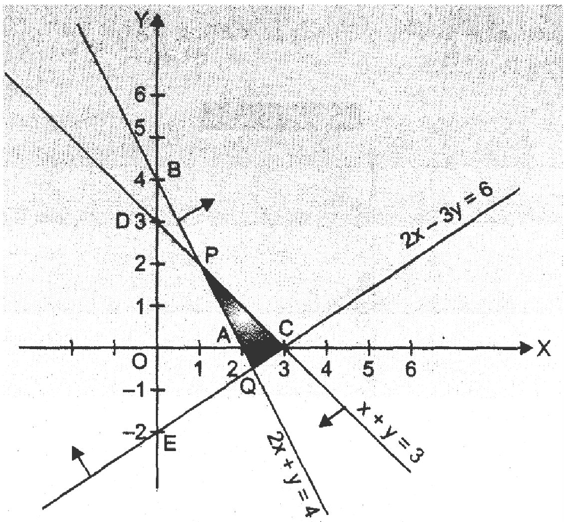
$$=\frac{(m+1)[2m+1+1-1](2m+2+1)}{3}$$

$$=\frac{(m+1)[2(m+1)-1][2(m+1)+1]}{3}$$

Thus P(m + 1) is true

41. **Ans:** Given system of inequalities is $2x+y\geq 4, x+y\leq 3,\ 2x-3y\leq 6$.

Graph of $2x+y\geq 4$ is that half of XOY – plane which lies above the line AB i.e. the line joining the points A(2, 0) and B(0, 4). Note that O(0, 0) lies in this region.



Graph of x + y ≤ 3 is that half of XOY-plane which .lies below the line CD i.e. the line joining the point C (3, 0) and D (0, 3). Note that O (0, 0) lies in this region.

Graph of 2x - 3y ≤ 6 is that half of XOY-plane which lies above the line EC i.e. the line joining the points E (0, - 2) and C (3, 0). Note that O (0, 0) lies in this region.

Graph of the given system of inequalities is the region common to all the three graphs and is shown shaded in the figure. it is bounded by the triangle CPQ.

42. **Ans:** i) Since the team will not include any girl, therefore, only boys are to be selected 5 boys out of 7 boys can be selected in ${}^7C_5$ ways. Therefore the required number of ways = ${}^7C_5 = 21$

ii) Since at least one boy and one girls are to be there in every team. Therefore, the team can consist of

a) 1 boy and 4 girls, b) 2 boys and 3 girls

c) 3 boys and 2 girls d) 4 boys and 1 girls

1 boy and 4 girls can be selected in ${}^7C_1 \times {}^4C_4$ ways

2 boy and 3 girls can be selected in ${}^7C_2 \times {}^4C_3$ ways

3 boy and 2 girls can be selected in ${}^7C_3 \times {}^4C_2$ ways

4 boy and 1 girl can be selected in ${}^7C_4 \times {}^4C_1$ ways

$\therefore$ Required number = ${}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140 = 441$

iii) Since, the team has to consist of at least 3 girls, the team can consists of

a) 3 girls and 2 boys or

b) 4 girls and 1 boy

3 girls and 2 boys can be selected in ${}^4C_3 \times {}^7C_2$ways

4 girls and 1 boys can be selected in ${}^4C_4 \times {}^7C_1$ways

Required = ${}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7 = 91$

43. **Ans:** The proof is obtained by applying principle of mathematical induction. Let the given statement be:

$P(n):(a+b)^n={}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+...+{}^nC_nb^n$

For n = 1 we have $P(1):(a+b)'={}^1C_0a+{}^1C_1a^0b=a+b$

Thus P(1) is true

Suppose p(m) is true for some positive integer,

i.e $(a+b)^m={}^mC_0a^m+{}^mC_1a^{m-1}b+{}^mC_2a^{m-2}b^2+...+{}^mC_mb^m$

we hall prove that P(m + 1) is also true i.e.,

Now, $(a+b)^{k+1}=(a+b)(a+b)^m$

$=(a+b)\left\{{}^mC_0a^m+{}^mC_1a^{m-1}b+{}^mC_2a^{m-2}b^2+...+{}^mC_mb^m\right\}$

$={}^mC_0a^{m+1}+{}^mC_1a^mb+{}^mC_2a^{m-1}b^2+...+{}^mC_mab^m+{}^mC_0a^mb+{}^mC_1a^{m-1}b^2+{}^mC_2a^{m-2}b^3+...+{}^mC_mb^{m+1}$

$={}^mC_0a^{m+1}+\left({}^mC_1+{}^mC_0\right)a^mb+\left({}^mC_2+{}^mC_1\right)a^{m-1}b^2+\left({}^mC_3+{}^mC_2\right)a^{m-2}b^3+...+{}^mC_mb^{m+1}$

$={}^{m+1}C_0a^{m+1}+{}^{m+1}C_1a^mb+{}^{m+1}C_2a^{m-1}b^2+...+{}^{m+1}C_{m+}b^{m+1}$

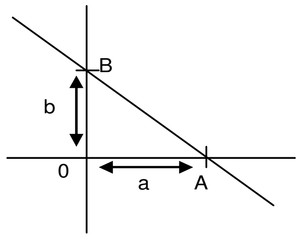
(By using ${}^{m+1}C_0=1, {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$ and ${}^nC_n=1={}^{m+1}C_{m+1}$)

$\therefore$ P(n) is true for n = m + 1

44. **Ans:** Consider a line which cuts the x and y axis at A and B respectively

Then OA = x-intercepts = a

OB = y- intercepts = b



$\therefore A=(A,0)=(x_1,y_1)$

$B=(0,b)=(x_2,y_2)$

w.k.t $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$

$\frac{x-a}{0-a}=\frac{y-0}{b-0}$

$\frac{x}{-a}-\frac{a}{-a}=\frac{y}{b}$

$\frac{-x}{a}+1=\frac{y}{b}\Rightarrow\frac{x}{a}+\frac{y}{b}=1$

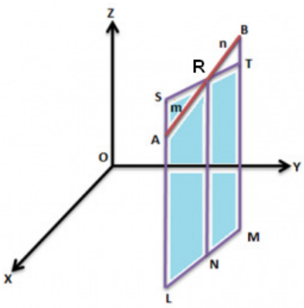
a = - 3, b = 2

$\frac{x}{-3}+\frac{y}{2}=1\Rightarrow\frac{-x}{3}+\frac{y}{2}=1$

$\Rightarrow -2x+3y=6$

$\Rightarrow 2x-3y=6$

45. **Ans:** Let th two given points be $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$. Let the point R(x, y, z) divide PQ in the given ratio m : n internally. Draw PL, QM and RN perpendicular to the XY-plane. Obviously PL II RN II QM and feet of these perpendiculars lie in a XY-plane. The points L, M and N will lie on a line which is the intersection of the plane containing PL, RN and QM with the XY-plane. Through the point R draw a line ST parallel to the line LM. Line ST will intersect the line LP externally at the point S and the line MQ at T, as shown in Fig



Also note that quadrilaterals LNRS and NMTR are parallelograms.

The triangles PSR and QTR ar Similar. Therefore,

$$\frac{m}{n}=\frac{PR}{QR}=\frac{SP}{QT}=\frac{SL-PL}{QM-TM}=\frac{NR-PL}{QM-NR}=\frac{z-z_1}{z_2-z}$$

This implies $z=\dfrac{mz_2+nz_1}{m+n}$

Similarity, by drawing perpendiculars to the XZ and YZ –planes, we get

$y=\dfrac{my_2+ny_1}{m+n}$ and $x=\dfrac{mx_2+nx_1}{m+n}$

Hence, the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n are

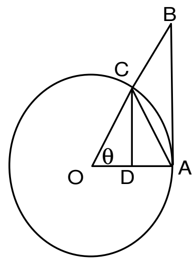
$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m-n}\right)$$

Mid point $=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2},\dfrac{z_1+z_2}{2}\right)$

$=\left(\dfrac{1+5}{2},\dfrac{2+6}{2},\dfrac{3+7}{2}\right)=(3,4,5)$

46. **Ans:** Proof we know that $\sin(-\theta)=-\sin\theta$ and $\cos(-\theta)=\cos\theta$. Hence, it is sufficient to prove th inequality for $0<\theta<\dfrac{\pi}{2}$

In the fig O is the centre of the unit circle such that the angle AOC is θ radians and $0<\theta<\dfrac{\pi}{2}$. Line segments BA and CD are perpendiculars to OA. Further, join AC.



Then

Area of ΔOAC < Area of sector OAC < Area of Δ OAB

i.e., $\dfrac{1}{2}OA.CD<\dfrac{\theta}{2\pi}.\pi(OA)^2<\dfrac{1}{2}QA.AB.$

i.e., CD < x. OA < AB

From ΔOCD,

$\sin\theta=\dfrac{CD}{OA}$ (since $OC=OA$) and hence $CD=OA\sin\theta$. Also $\tan\theta=\dfrac{AB}{OA}$ and

Hence $AB=OA.\tan x$, Thus $OA\sin\theta<OA\theta<OA\tan\theta$

Since length OA is positive, we have $\sin\theta<\theta<\tan\theta$

Since $0<\theta<\dfrac{\pi}{2}$, sin x is positive and thus by dividing throughout by sin x, we have $1<\dfrac{\theta}{\sin\theta}<\dfrac{1}{\cos\theta}$

Taking reciprocals throughout, we have

$$\cos\theta<\frac{\sin\theta}{\theta}<1$$

Taking limit as $\theta\to0$ throughout we get $\lim\limits_{\theta\to0}\cos\theta<\lim\limits_{\theta\to0}\dfrac{\sin\theta}{\theta}<1$

$$1<\lim_{\theta\to0}\frac{\sin\theta}{\theta}=1$$

∵ there is no real number between 1 and 1

$$\lim_{\theta\to0}\frac{\tan\theta}{\theta}=\lim_{\theta\to0}\frac{\sin\theta}{\theta\cos\theta}$$

$$=\frac{\lim\limits_{\theta\to0}\dfrac{\sin\theta}{\theta}}{\lim\limits_{\theta\to0}\cos\theta}=\frac{1}{1}=1$$

47. **Ans:**

Class	f_i	c-f	x_i	$\|x_i-M\|$	$f\|x_i-M\|$
0-10	6	6	5	22.857	137.142
10-20	8	14	15	12.857	102.856
20-30	14	28	25	2.857	39.998
30-40	16	44	35	7.143	114.288
40-50	4	48	45	17.143	68.572
50-60	2	50	55	27.143	54.286
	N = 50				$\sum f\|x_i - M\| = 517.142$

$\frac{n}{2} = \frac{50}{2} = 25$

l = 20

C= 14, f = 14, n = 10

$\therefore$ class median is 20 – 30

$$M = l + \left(\frac{\frac{N}{2} - C}{f}\right) = 20 + \left(\frac{25-14}{14}\right) \times 10$$

$$= 20 + \frac{110}{14} = 20 + 7.857$$

$$= 27.857$$

Part – E

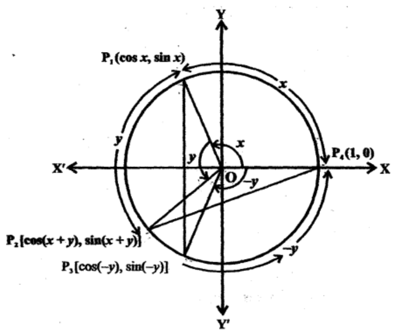
VI. Answer any One of the following: **1 x 10 = 10**

48. **Ans:** a) $\cos 15^0 = \cos(45^0 - 30) = \cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

cos (x + y) = cos x cos y – sin x sin y

Consider the unit circle with centre at the origin. Let X be the angle P_4OP_1 and y be the angle P_1OP_2. Then (x + y) is the angle P_4OP_2 Also let (- y) be the angle P_4OP_3 Therefore, P_1. P_2. P_3 and P_4 will have the coordinates P_1(cos x, sin x), P_2[cos(x + y), sin (x + y)], P_3[cos (- y), sin(- y)1 and P_4(1, 0)



Consider the triangles P_1OP_3 and P_2OP_4. They are congruent (Why?). Therefore, P_1P_3 and P_2P_4 are equal. By using distance formula, we get

$P_1P_3^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(y)]^2$

$= (\cos x - \cos y)^2 + (\sin x + \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y + 2\sin x \sin y$

$= 2 - 2(\cos x \cos y - \sin x \sin y)$ (Why?)

Also, $P_2P_4^2 = [1 - \cos(+y)]^2 + [0 - \sin(x + y)]^2$

$= 1 - 2\cos(x + y) + \cos^2(x + y) + \sin^2(x + y)$

$= 2 - 2\cos(x + y)$

Since $P_1P_3 = P_2P_4$, we have $P_1P_3^2 = P_2P_4^2$

Therefore, 2 -2 (cos x cos y – sin x sin y) = 2-2 cos(x + y)

Hence cos(x + y) = cos x cos y +sin x, sin y

b) $S = 7 + 77 + 777 + 7777 + 77777 + ...n$ terms

$= 7\left[1 + 11 + 111 + 1111 + 11111 + ...n \text{ terms}\right]$

$= \frac{7}{9}\left[9 + 99 + 999 + 9999 + ...n \text{ terms}\right]$

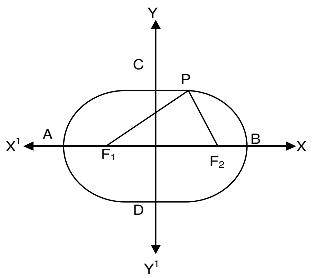
$= \frac{7}{9}\left[10 - 1 + 10^2 - 1 + 10^3 - 1 + 10^4 - 1 + ...n \text{ terms}\right]$

$= \frac{7}{9}\left[10 + 10^2 + 10^3 + 10^4 + ...n \text{ term} - (1 + 1 + 1 + ... n \text{terms}\right]$

$= \frac{7}{9}\left\{\frac{10(10^n - 1)}{10 - 1} - n\right\}$

$= \frac{7}{9}\left\{\frac{10}{9}\left(10^n - 1\right) - n\right\}$

49. **Ans:** a) An ellipse is the set of all point in a plane, the sum of whose distances from two fixed points in the plane is a constant.



Let F_1 and F_2 be the foci and O be the midpoint of the line segment F_1F_2. Let O be the origin and the line from O through F_2 be the positive x- axis and that through F_1 be the negative x-axis. let the line through O perpendicular to the x-axis be the y-axis. Let the co-ordinate of F_1 be (-c,o) and F2 be (c, 0)

Let p(x, y) be any point on the ellipse such that the sum of the distance from P to the two foci be 2a

$PF_1 + PF_2 = 2a$

$\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$

$\Rightarrow \sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$

$(x+c)^2 + \cancel{y^2} = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + (x-c)^2 + \cancel{y^2}$ (squary on both sides)

$\cancel{x^2} + 2cx + \cancel{c^2} = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + \cancel{x^2} - 2cx + \cancel{c^2}$

$\cancel{4}cx = \cancel{4}a\left[a - \sqrt{(x-c)^2+y^2}\right]$

$cx = a^2 - a\sqrt{(x-c)^2+y^2}$ $(cx-a^2)^2 = [a\sqrt{(x-c)^2+y^2}]^2$ squary on both sides

$c^2x^2 - 2cxa^2 + a^4 = a^2\left[(x-c)^2+y^2\right]$

$c^2x^2 - 2\cancel{cxa^2} + a^4 = a^2x^2 - 2\cancel{cxa^2} + a^2c^2 + a^2y^2$

$a^4 - a^2c^2 = a^2x^2 - c^2x^2 + a^2y^2$

$a^2(a^2-c^2) = x^2(a^2-c^2) + a^2y^2 \div a^2(a^2-c^2)$

$1 = \dfrac{x^2}{a^2} + \dfrac{y^2}{a^2-c^2}$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where $b^2 = a^2 - c^2$

b) When <1

$f(x) = ax^2 - 3x + 4$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} ax^2 - 3x + 4$

$= a - 3 + 4$

When $x > 1$; $f(x) = bx + 5$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} bx + 5$

$= b + 5$

At $x = 1$

$f(x) = 3$

$f(1) = 3$

$\lim\limits_{x\to 1} f(x)$ exist if $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\therefore a - 3 + 4 = b + 5$

Given $\lim\limits_{x\to 1} f(xx) = f(1)$

$a - 3 + 4 = b + 5 = 3$

$a - 3 + 4 = 3 \quad b + 5 = 3$

$a = 6 - 4 \qquad \boxed{b = -2}$

$\boxed{a = 2}$