MODEL PAPER - 1
MATHEMATICS

I PUC
Time: 3 Hrs **Max Marks: 100**

Part – A

I. Answer the following: **1 x 10 = 10**

1. **Ans:** A = [0, 7)
2. **Ans:** $x-1=2$ $\quad$ $y+3=x+4$

   X = 3 $\quad$ y = 4
3. **Ans:** $240^0 = \cancel{240}^{4} \times \frac{\pi}{\cancel{180}_{3}} = \frac{4\pi^c}{3}$
4. **Ans:** 1 – I - (-1 + 6i) = 1 – i + 1 – 6i = 2 – 7i
5. **Ans:** a = $\frac{5}{2}$, $r = \frac{1}{2}$ $\quad n = 20$ WKT $T_n = ar^{n-1}$

   $$T_{20} = \left(\frac{5}{2}\right)\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2^{20}}$$
6. **Ans:** $a_n = \frac{n-3}{4}$

   $a_1 = \frac{-1}{2}$, $a_2 = \frac{-1}{4}$, $a_3 = 0$
7. **Ans:** Slope = 1
8. **Ans:** f(x) = $2x - \frac{3}{4}$

   $f'(x) = 2$
9. **Ans:** The number 2 is not greater than 7
10. **Ans:** S = {HHH , HHT, HTH, HTT, THH, THT, TTH, TTT}, n(s) = 8

    A = Exactly one head

    A = {HTT, THT, TTH}, n(A) = 3

    $p(A) = \frac{3}{8}$

Part – B

II. Answer any TEN of the following: **10 x 2 = 20**

11. **Ans:** $A' = \{1,3,5,7,9\}$ $\quad$ $A' \cap B' = \{1,9\}$

    $B' = \{1,4,6,8,9\}$

    $A \cup B = \{2,3,4,5,6,7,8\}$

    $(A \cup B)' = \{1,9\}$

    $\therefore (A \cup B)' = A' \cap B'$
12. **Ans:** $n(x \cap y) = n(x) + n(y) - n(x \cup y)$

    $= 17 + 23 - 3 = 37$
13. **Ans:** $B \cap C = \{4\}$

    $Ax(B \cap C) = \{(1,4),(2,4),(3,4)\}$

14. **Ans:** $\sin 75^0 = \sin(45^0 + 30^0) = \sin 45^0 \cos 30^0 + \cos 45^0 \sin 30^0$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

15. **Ans:** $2\cos^2 x - 3\sin x = 0$

$2(1 - \sin^2) - 3\sin x = 0$

$2 - 2\sin^2 x - 3\sin x = 0$

$2\sin^2 x + 3\sin - 2 = 0$

$2\sin^2 x + 4\sin x - \sin x - 2 = 0$

$2\sin x(\sin x + 2) - (\sin x + 2) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -2$ No solution

$x = n\pi + (-1)^n \frac{\pi}{3}$ $, \forall n \in I$

16. **Ans:** $\frac{1+3i}{1-2i} = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{1+2i+3i+6i^2}{1+4}$

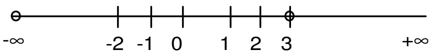
$$= \frac{-5+5i}{5} = -1 + i$$

17. **Ans:** $7x + 3 < 5x + 9$

$7x - 5x + \cancel{3} - \cancel{3} + < \cancel{5x} - \cancel{5x} + 9 - 3$

$2x < 6$

$x < 3$



18. **Ans:** $m_1 = -\sqrt{3}$, $\qquad m_2 = \frac{-1}{\sqrt{3}}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1+1}\right|$$

$$= \left|\frac{-3+1}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}^{C}$$

19. **Ans:** Equation of a line parallel to 3x – 4y + 2 = 0 is of the form 3x – 4y + k = 0 …(1)

(1) passes through (-2, 3) i.e,. put x = -2, y = 3

$-6 - 12 + k = 0$

$K = 18$

Sub K in (1)

$3x - 4y + 18 = 0$

20. **Ans:** Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$

$$= \sqrt{25 + 9 + 9} = \sqrt{43}$$

21. **Ans:** $\underset{x \to 3}{\text{lt}} \frac{x^4 - 81}{2x^2 - 5x - 3} = \underset{x \to 3}{\text{lt}} \frac{x^4 - 3^4}{(x-3)(2x+1)} = \frac{4(3^{4-1})}{6+1} = \frac{4 \times 3^3}{7}$

$$= \frac{36}{7}$$

22. **Ans:** Converse: if a parallelogram is a rhombus then it is a square
Contraposite: If a parallelogram is not a rhombus then it is not a square

23. **Ans:** $C.V = 70,\ \sigma = 16\ \ \bar{x} = ?$

w.k.t $\quad C.V = \frac{\sigma}{\bar{x}} \times 100$

$\bar{x} = \frac{\sigma}{CV} \times 100$

$= \frac{16}{7\cancel{0}} \times 10\cancel{0} = \frac{160}{7}$

24. **Ans:** $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

n(s) = 8

E = atleast two heads

$= \{HHH, HHT, HTH, THH\}$

n(E)=4

$P(E) = \frac{n(E)}{n(s)} = \frac{1}{2}$

Part – C

III. Answer any TEN of the following: 10 x 3 = 30

25. **Ans:** Let x be the students who like to play cricket and y be the students who like to play football

$n(x) = 24,\ n(y) = 16,\ n(x \cup y) = 35,\ n(x \cap y) = ?$

w.k.t $n(x \cap y) = n(x) + n(y) - n(x \cup y)$

= 24 + 16 – 35 = 5

$\therefore$ 5 students like to play both games

26. **Ans:** a) $(f+g)(x) = f(x) + g(x) = x^2 + 2x + 1$

b) $(f-g)(x) = f(x) - g(x) = x^2 - (2x+1) = x^2 - 2x - 1$

c) $(fg)(X) = (f(x))(g(x)) = x^2(2x+1) = 2x^3 + x^2$

27. **Ans:** x and y lies in second quadrant

cos & sec are negative, remaining are negative

$\sin x = \frac{3}{5}\ ; \cos x = \frac{-4}{5}$

$\cos y = \frac{-12}{13}\ ; \sin y = \frac{5}{13}$

$\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{3}{5}\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65}$

28. **Ans:** $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{6+9i-4i-6i^2}{2-i+4i+2} = \frac{12+5i}{4+3i}$

$= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i} = \frac{48-36i+20i+15}{16+9}$

$= \frac{63-16i}{25} = \frac{63}{25} - i\frac{16}{25}$

Conjugate in $\frac{63}{25} + i\frac{16}{25}$

29. **Ans:** $x^2+3+9=0$

$a=1,\ b=3,\ c=9$

W.K.T $b^2-4ac=9-36=-27<0$

$$x=\frac{-b\pm i\sqrt{4ac-b^2}}{2a}$$

$$x=\frac{-3\pm i3\sqrt{3}}{2}$$

30. **Ans:** There are 12 letters of which N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different

$\therefore$ The required permutation $=\frac{12!}{3!4!2!}=1663200$

There are 5 vowels in the given word, which 4E's and 1I. Since, they have to always occur together, we treat them as a single object $\boxed{\text{EEEEI}}$ for the time being. This single object together with 7 remaining objects will account for 8 objects. These 8 objects in which thre are 3N's and 2D's , can be arranged in $\frac{8!}{3!2!}$ ways

Corresponding to each of these arrangements, the 5 vowels EEEE and I can be rearranged in $\frac{5!}{4!}$ ways

$\therefore$ the required permutation = $\frac{8!}{3!2!}\times\frac{5!}{4!}=16800$

31. **Ans:** $(\sqrt{3}+\sqrt{2})^4={}^4C_0(\sqrt{3})^4+{}^4C_1(\sqrt{3})^3\sqrt{2}+{}^4C_2(\sqrt{3})^2(\sqrt{2})^2+{}^4C_3(\sqrt{3})(\sqrt{2})^3+{}^4C_4(\sqrt{2})^4$

${}^4C_0=1={}^4C_4$; ${}^4C_1=4={}^4C_3$

$${}^4C_2=\frac{4!}{2!2!}=\frac{{}^{2}\cancel{4}\times3\times\cancel{2!}}{\cancel{2}\times\cancel{2}}=6$$

$(\sqrt{3}+\sqrt{2})^4=3^2+6\times3\sqrt{6}+4\times3\times2+4\times2\times\sqrt{6}+2^2$

$=9+18\sqrt{6}+24+8\sqrt{6}+4$

$(\sqrt{3}-\sqrt{2})^4=9-18\sqrt{6}+24-8\sqrt{6}+4$

$\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=36\sqrt{6}+16\sqrt{6}=52\sqrt{6}$

32. **Ans:** 105, 110, 115,….995

$a=105\ \ d=5,\ T_n=995,\ n=?$

$T_n=a+(n-1)d$

995 = 105 +(n – 1) d

$\frac{890}{5}=n-1\Rightarrow n=178+1$

n = 179

W.K.T

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_{179}=\frac{179}{2}[2\times105+178\times5]$$

$$=\frac{179}{2}[1100]=179\times550$$

$=98450$

33. **Ans:** a = -6, d = $\frac{-11}{2}+6=\frac{1}{2}$

$S_n=-25$

w.k.t $s_n=\frac{n}{2}[2a+(n-1)d]$

$-25=\frac{n}{2}\left[-12+\frac{n-1}{2}\right]$

$-50=-12n+\frac{n(n-1)}{2}\Rightarrow -100=-24n+n^2-n$

$n^2-25n+100=0\Rightarrow n^2-20n-5n+100=0$

n(n - 20) -5(n - 20) = 0 $\quad\therefore n=5$ or $n=20$

34. **Ans:** $x^2=-16y$

a = 4

focus (0, -a) = (0, -4)

Directrix : y = a $\Rightarrow y=4$

LLR = 4a = 16

35. **Ans:** f(x) = sinx

$f(x+\Delta x)=\sin(x+\Delta x)$

w.k.t

$\frac{d}{dx}[f(x)]=\underset{\Delta x\to 0}{\text{lt}}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

$\frac{d(\sin x)}{dx}=\underset{\Delta x\to 0}{\text{lt}}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$

$=\underset{\Delta x\to 0}{\text{lt}}\frac{2\cos\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{\cancel{x}+\Delta x-\cancel{x}}{2}\right)}{\Delta x}$

$=\underset{\Delta x\to 0}{\text{lt}}\frac{{}^{1}\cancel{2}\cos\left(\frac{2x+\Delta x}{2}\right).\sin\left(\frac{\Delta x}{2}\right)}{\cancel{2}\times\frac{\Delta x}{2}}$

$=\cos\left(\frac{2x+0}{2}\right)=\cos x$

36. **Ans:** In this method, we assume that the given statement is false. That is we assume $\sqrt{2}$ is rational.

This means that there exits positive integers a and b such that $\sqrt{2}=\frac{a}{b}$, where a and b have no common factors.

$2=\frac{a^2}{b^2}\Rightarrow a^2=2b^2\Rightarrow 2$ divides a

$\therefore$ there exist an integer c such that a = 2c

then $a^2=4c^2$ & $a^2=2b^2$

Hence $2b^2=4c^2\Rightarrow b^2=2c^2\Rightarrow$ 2 divides b

But we have already shown that 2 division a. This implies that 2 in a common factor of both a and b which contradicts our earlier assumption that a and b have no common factors

This shows that the assumption $\sqrt{2}$ is irrational is wrong

Hence the statement $\sqrt{2}$ is irrational is true

37. **Ans:** Total number of persons = 2 + 2 = 4 out of these four person, two can be selected in ${}^4C_2$ ways

(i) No man in the committee of two means there will be two women in the committee. Out of two women two can be selected in ${}^2C_2 = 1$ way

$\therefore$ p(no man) = $\frac{{}^2C_2}{{}^4C_2} = \frac{1\times2\times1}{4\times3} = \frac{1}{6}$

(ii) One man in the committee means that there is one woman

One man out of 2 can be selected in ${}^2C_1$ ways and

One women out of 3 can be selected in ${}^2C_1$ ways

Together they can be selected in ${}^2C_1 \times {}^2C_1$ ways

$\therefore \text{p(one man)} = \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} = \frac{2\times2}{2\times3} = \frac{2}{3}$

(iii) Two men can be selected in ${}^2C_2$ ways

$\text{p(two men)} = \frac{{}^2C_2}{{}^4C_2} = \frac{2}{{}^4C_2} = \frac{1}{6}$

38. **Ans:** S = $\{HH, HT, TH, TT\}\, n(s) = 4$

E = at-least one tail = {HH, HT, TH}

n(E) = 3

p(E) = $\frac{3}{4}$

Part – D

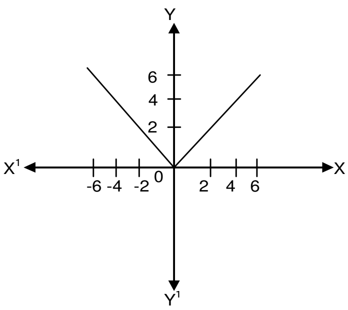
IV. Answer any Six of the following: **6 x 5 = 30**

39. **Ans:** The function f:R $\to$R defined by f(x) = $|x|$ for each $x \in R$ is called modulus function

i.e., $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain = R

Range = Positive real numbers including zero



40. **Ans:** tan 4x = tan 2(2x) = $\frac{2\tan 2x}{1-\tan^2 2x}$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2} = \frac{\frac{4\tan x}{\cancel{1-\tan^2 x}}}{\frac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^{\cancel{2}}}}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x + \tan^4 x}$$

41. **Ans:** $P(n):1^3+2^3+3^3+...+n^3=\dfrac{n^2(n+1)^2}{4}$

Let n = 1

LHS = $n^3=1^3=1$

RHS = $\dfrac{n^2(n+1)^2}{4}=\dfrac{1^2(1+1)^2}{4}=\dfrac{4}{4}=1$

$\therefore$ LHS = RHS

$\therefore P(n)$ is true for n = 1

Assume P(n) is true for n = m

$P(m):1^3+2^3+3^3+...+m^3=\dfrac{m^2(m+1)^2}{4}$

Adding $(m+1)^3$ on both sides

$p(m+1):1^3+2^3+3^3+...+m^3+(m+1)^3=\dfrac{m^2(m+1)^2}{4}+(m+1)^3$

$=(m+1)^2\left[\dfrac{m^2}{4}+m+1\right]$

$=(m+1)^2\left[\dfrac{m^2+4m+4}{4}\right]$

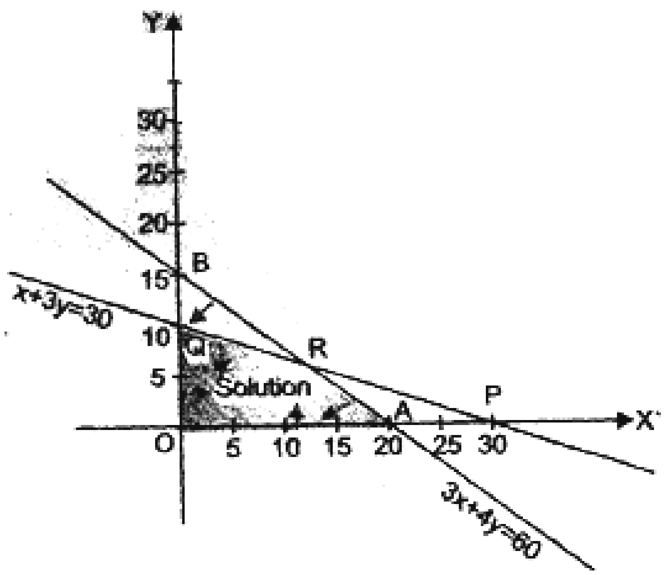
$=\dfrac{(m+1)^2(m+2)^2}{4}$

$=\dfrac{(m+1)[(m+1)+1]^2}{4}$

$\therefore p(n)$ is true for n = m + 1

42. **Ans:** Given system of inequalities is $3x+4y\le 60, x+3y\le 30, x\ge 0, y\ge 0$. The inequalities $x\ge 0, y\ge 0$ taken together represent the first quadrant.

Graph 3x + 4y ≤ 60 is that half of XOY – plane which lies below the line AB i.e., the line joining the points A(20, 0) and B(0,1) Note that O(0, 0) lies in this region.



Graph of x + 3y ≤ 30 is that half of XOY- plane which lis below the line PQ i.e., the line joining the points P(30, 0) and Q(0, 10). Note that O(0, 0) lies in this region.

The graph of the given system of inequalities is the region common to all the graph which is shown shads in the figure. it is bounded by the quadrilateral OARQ.

43. **Ans:** $2^nC_3 : {}^nC_3 = 12:1$

$$\frac{\frac{2n!}{\cancel{3}!(2n-3)!}}{\frac{n!}{\cancel{3}!(n-3)!}} = 12$$

$$\frac{2\cancel{n}(2n-1)(2n-2)\cancel{(2n-3)}!\,\cancel{(n-3)}!}{\cancel{(2n-3)}!\,\cancel{n}(n-1)(n-2)\cancel{(n-3)}!} = 12$$

$$\frac{\cancel{2}(2n-1)\cancel{2}\,\cancel{(n-1)}}{\cancel{(n-1)}(n-2)}\cancel{12}^3 = \cancel{12}^3$$

$2n-1 = 3n-6$

$-1+6 = 3n-2n$

n = 5

44. **Ans:** For any positive integer n

$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \dots + {}^nC_na^n$

Proof; The proof is by mathematical statement

$P(n): (x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \dots. + {}^nC_na^n$

For n = 1

$P(1): (x+a)^1 = {}^1C_0x^1 + {}^1C_1x^0a = x+a$

Thus p(1) is true

Assume p(n) is true for n = m

$p(m): (x+a)^m = {}^mC_0x^m + {}^mC_1x^{m-1}a + {}^mC_2x^{m-2}a^2 + \dots + {}^mC_ma^m$

We shall prove that p(m + 1) is also true i.e

$(x+a)^{m+1} = (x+a)(x+a)^m$

$= (x+a)^{m+1} = (x+a)(x+a)^m = (x+a)\{{}^mC_0x^m + {}^mC_1x^{m-1}a + {}^mC_2x^{m-2}a^2 + \dots + {}^mC_na^m\}$

$= {}^mC_0x^{m+1} + {}^mC_1x^ma + {}^mC_2x^{m-1}a^2 + \dots + {}^mC_ma^mx + {}^mC_0x^ma + {}^mC_1x^{m-1}a^2 + {}^mC_2x^{m-2}a^3 + \dots + {}^mC_ma^{m+n}$

$= {}^mC_0x^{m+1} + ({}^mC_1 + {}^mC_0)x^ma + ({}^mC_2 + {}^mC_1)x^{m-1}a^2 + \dots + ({}^mC_m + {}^mC_{m-1})x\,a^m + {}^mC_ma^{m+1}$

${}^mC_0 = {}^{m+1}C_0 = 1$; ${}^mC_m = {}^{m+1}C_{m+1} = 1$

W.K.T ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^mC_1 + {}^mC_0 = {}^{m+1}C_1$; ${}^mC_2 + {}^mC_1 = {}^{m+1}C_2$

${}^mC_3 + {}^mC_2 = {}^{m+1}C_3 \dots {}^mC_m + {}^{m+1}C_3 \dots. {}^mC_m + {}^mC_{m-1} = {}^{m+1}C_m$

$(x+a)^{m+1} = {}^{m+1}C_0x^{m+1} + {}^{m+1}C_1x^ma + {}^{m+1}C_2x^{m-1}a^2 + \dots + {}^{m+1}C_{m-1}x\,a^m + {}^{m+1}C_{m+1}a^{m+1}$

Hence, $\therefore p(n)$ is true for n = m + 1

45. **Ans:** Equation of the line whose intercepts on a and b is

$\frac{x}{a} + \frac{y}{b} = 1$ or $bx + ay - ab = 0$

P = length of $\perp^r$ from origin upon the line bx + ay – ab = 0

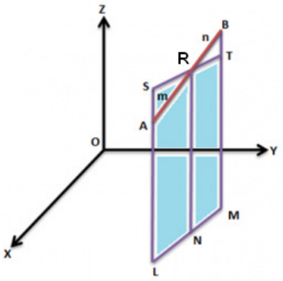
$$p = \left|\frac{b(0)+a(0)-ab}{\sqrt{a^2+b^2}}\right|$$

$$p^2 = \frac{a^2b^2}{a^2+b^2}$$

$$\frac{a^2+b^2}{a^2b^2} + = \frac{1}{p^2} \Rightarrow \frac{\cancel{a}^2}{\cancel{a}^2b^2} + \frac{\cancel{b}^2}{a^2\cancel{b}^2} = \frac{1}{p^2}$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$$

46. **Ans:** Let th two given points be $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$. Let the point R(x, y, z) divide PQ in the given ratio m : n internally. Draw PL, QM and RN perpendicular to the XY-plane. Obviously PL II RN II QM and feet of these perpendiculars lie in a XY-plane. The points L, M and N will lie on a line which is the intersection of the plane containing PL, RN and QM with the XY-plane. Through the point R draw a line ST parallel to the line LM. Line ST will intersect the line LP externally at the point S and the line MQ at T, as shown in Fig



Also note that quadrilaterals LNRS and NMTR are parallelograms.
The triangles PSR and QTR ar Similar. Therefore,

$$\frac{m}{n} = \frac{PR}{QR} = \frac{SP}{QT} = \frac{SL - PL}{QM - TM} = \frac{NR - PL}{QM - NR} = \frac{z - z_1}{z_2 - z}$$

This implies $\quad z = \dfrac{mz_2 + nz_1}{m + n}$

Similarity, by drawing perpendiculars to the XZ and YZ –planes, we get

$y = \dfrac{my_2 + ny_1}{m + n}$ and $x = \dfrac{mx_2 + nx_1}{m + n}$

Hence, the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n are

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m - n}\right)$$

Mid point $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$

$$= \left(\frac{1+5}{2}, \frac{2+6}{2}, \frac{3+7}{2}\right) = (3,4,5)$$

47. **Ans:** $\lim\limits_{\theta \to 0} \dfrac{\tan\theta}{\theta} = \lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta\cos\theta}$

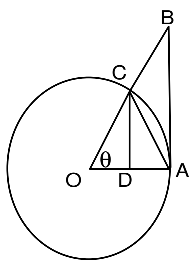
$= \dfrac{\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta}}{\lim\limits_{\theta \to 0} \cos\theta} = \dfrac{1}{1} = 1$ Proof we know that $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$. Hence, it is sufficient to prove the inequality for $0 < \theta < \frac{\pi}{2}$

In the fig O is the centre of the unit circle such that the angle AOC is θ radians and $0 < \theta < \dfrac{\pi}{2}$. Line segments BA and CD are perpendiculars to OA. Further, join AC. Then

Area of ΔOAC < Area of sector OAC < Area of Δ OAB

i.e.,

$$\frac{1}{2}(OA).(CD) < \frac{\theta}{2\pi}.\pi(OA)^2 < \frac{1}{2}(QA).(AB).$$



i.e., CD < x. OA < AB

From ΔOCD,

$\sin\ \theta = \frac{CD}{OA}$ (since $OC = OA$) and hence $CD = OA\sin\theta$. Also $\tan\theta = \frac{AB}{OA}$ and

Hence $AB = OA.\tan x$, Thus $OA\sin\theta < OA\theta < OA\tan\theta$

Since length OA is positive, we have $\sin\theta < \theta < \tan\theta$

Since $0 < \theta < \frac{\pi}{2}$, sin x is positive and thus by dividing throughout by sin θ, we have $1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$

Taking reciprocals throughout, we have

$$\cos\theta < \frac{\sin\theta}{\theta} < 1$$

Taking limit as $\theta \to 0$ throughout we get $\lim_{\theta\to 0}\cos\theta < \lim_{\theta\to 0}\frac{\sin\theta}{\theta} < 1$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

($\because$ there is no real number between 1 and 1)

48. **Ans:** We make the following Table from the given data:

Marks obtained	**Number of students f_i**	**Mid- points**	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
10-20	2	15	30	30	60
20-30	3	25	75	20	60
30-40	8	35	280	10	80
40-50	14	45	630	0	0
50-60	8	55	440	10	80
60-70	3	65	195	20	60
70-80	2	75	150	30	60
	40		**1800**		**400**

Here $N = \sum_{i=1}^{7} f_i = 40$, $\sum_{i=1}^{7} f_ix_i = 1800$, $\sum_{i=1}^{7} f_i\lvert x_i - \bar{x}\rvert = 400$

Therefore $\bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_ix_i = \frac{1800}{40} = 45$

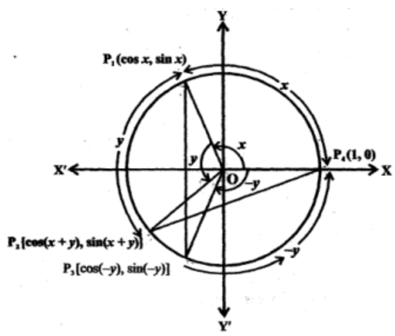
and $M.D(\bar{x}) = \frac{1}{4}\sum_{i=1}^{7} f_i\lvert x_i - \bar{x}\rvert = \frac{1}{40}\times 400 = 10$

Part – E

VI. Answer any One of the following: **1 x 10 = 10**

49. **Ans:** a) cos (x + y) = cos x cos y – sin x sin y

Consider the unit circle with centre at the origin. Let X be the angle P_4OP_1 and y be the angle P_1OP_2. Then (x + y) is the angle P_4OP_2 Also let (- y) be the angle P_4OP_3 Therefore, P_1. P_2. P_3 and P_4 will have the coordinates P_1(cos x, sin x), P_2[cos(x + y), sin (x + y)], P_3[cos (- y), sin(- y)1 and P_4(1, 0)



Consider the triangles P_1OP_3 and P_2OP_4. They are congruent (Why?). Therefore, P_1P_3 and P_2P_4 are equal. By using distance formula, we get

$$P_1P_3^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(-y)]^2$$

$$= (\cos x - \cos y)^2 + (\sin x + \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y + 2\sin x \sin y$$

$$= 2 - 2(\cos x \cos y - \sin x \sin y) \qquad (\text{Why?})$$

Also, $P_2P_4^2 = [1 - \cos(x+y)]^2 + [0 - \sin(x+y)]^2$

$$= 1 - 2\cos(x+y) + \cos^2(x+y) + \sin^2(x+y)$$

$$= 2 - 2\cos(x+y)$$

Since $P_1P_3 = P_2P_4$, we have $P_1P_3^2 = P_2P_4^2$

Therefore, 2 -2 (cos x cos y – sin x sin y) = 2-2 cos(x + y)

Hence cos(x + y) = cos x cos y +sin x, sin y

b) Given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$

Let T_n denote the nth term, then

$$T_n = 1^2 + 2^2 + 3^2 + \text{upto n terms} = \sum n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \qquad T_n = \frac{1}{6}(2n^3 + 3n^2 + n)$$

$$\Rightarrow \qquad S_n = \frac{1}{6}\{2\sum n^3 + 3\sum n^2 + \sum n\}$$

$$= \frac{1}{6}\left\{2\left\{\frac{n^2(n+1)^2}{4}\right\} + 3\left\{\frac{n(n+1)(2n+1)}{6}\right\} + \frac{n(n+1)}{2}\right\}$$

$$= \frac{1}{6}\left\{\frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2}\right\}$$

$$= \frac{n(n+1)}{6\times 2}\{n(n+1) + 2n + 1 + 1\}$$

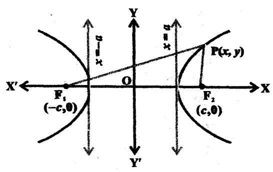
$$= \frac{n(n+1)}{12}\{n^2 + 3n + 2\}$$

$$= \frac{n(n+1)(n+1)(n+2)}{12} = \frac{n(n+1)(n+2)}{12}$$

50. **Ans:** a) A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

Let F_1 and F_2 be the foci and O be the mid-point of the line segment F_1F_2. Let O be the origin and the line through O through F_2 be the positive x-axis and that through F_1 as the negative x-axis. The line through O perpendicular to the x-axis be the y-axis. Let the coordinates of F_1 be (- c,0) and F_2 be (c,0).

Let P(x, y) be any point on the hyperbola such that the difference of the distances from P to the farther point minus the closer point be 2a.



So given, $PF_1 - PF_2 = 2a$

Using the distance formula, we have

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

i.e., $\sqrt{(x+c)^2+y^2} = 2a + \sqrt{(x-c)^2+y^2}$

Squaring both side, we get

$(x+c)^2+y^2 = 4a^2 + 4a\sqrt{(x-c)^2+y^2} + (x-c)^2+y^2$

and on simplifying, we get

$\frac{cx}{a} - a = \sqrt{(x-c)^2+y^2}$

On squaring again and further simplifying. We get

$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$

i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (since $c^2 - a^2 = b^2$)

Hence any point on the hyperbola satisfies $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

b) $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \frac{x^{98}}{98} + ... + \frac{x^2}{2} + x + 1$

$f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \frac{98x^{97}}{98} + ... + \frac{2x}{2} + 1$

$= x^{99} + x^{98} + x^{97} + ... + x + 1$

$f'(o) = 0 + 0 + 0 + ... + 0 + 1$

$f'(0) = 1$

$f'(1) = 1^{99} + 1^{98} + 1^{97} + ... + 1 + 1$ (100 times)

= 100

$\therefore f'(1) = 100.1$

$f'(1) = 100 f'(0)$